COMPLEXITY: Exercise No. 11
due next lesson

1. (Test 94)Consider the following problem:
   Input: An undirected graph G.
   Goal: Find a minimum number of vertex disjoint cycles in G such that each vertex in G is contained in exactly one cycle.
   Prove that if $\mathbf{P} \neq \mathbf{NP}$ then there is no absolute approximation algorithm for this problem.

2. Consider a minimization problem whose decision problem is strongly **NP**-complete, $\mathrm{OPT}(I)$ is a positive integer for any instance I, and for some polynomial p, $\mathrm{OPT}(I) \leq p(\mathrm{NUM}(I))$ for any instance I ($\mathrm{NUM}(I)$ is the sum of the numbers appearing in the instance I). Show that the problem has a fully polynomial-time approximation scheme iff $\mathbf{P} = \mathbf{NP}$.

3. Show that the following problem is fixed parameter tractable:
   WEIGHTED 3-SAT:
   Instance: An 3-CNF formula Φ, a positive integer K.
   Question: Is there a satisfying assignment for Φ in which the number of variable with TRUE value is at most K?

4. Prove that the following generalization of *Minimum Circuit* is also in $\Pi_2\mathbf{P}$:
   Instance: A Boolean circuit C, and integer k with $k \leq |C|$.
   Question: Are there no more than k circuits, each with fewer gates than C, computing the same function ? (i.e. C is one of the $k+1$ minimum circuits)

5. Consider the following reasoning:

   (a) $QBF \in \mathbf{PSPACE}$

   (b) There exists k such that $QBF \in \mathbf{SPACE}(n^k)$.

   (c) QBF is **PSPACE**-complete.

   (d) Any $L \in \mathbf{PSPACE}$ is reducible to QBF.

   (e) Any $L \in \mathbf{PSPACE}$ can be reduced, in logarithmic space, to QBF, and solved in n^k space.

   (f) $\mathbf{PSPACE} = \mathbf{SPACE}(n^k)$

   (g) But $\mathbf{SPACE}(n^k) \neq \mathbf{SPACE}(n^k \log(n^k))$ (Space Hierarchy Theorem), a contradiction !

   Explain the “bug”.