

**ADVANCED NUMBER THEORY  
ASSIGNMENT 1  
DUE DATE: WEDNESDAY, MAY 28, 2008**

1. Show that

$$\sum_{n \leq x} \log n = x \log x - x + O(\log x)$$

2. Show that there is a constant  $C$  so that

$$\sum_{n=1}^N \frac{1}{n} = \log N + C + O\left(\frac{1}{N}\right)$$

3. The von Mangoldt function  $\Lambda(n)$  is defined as

$$\Lambda(n) = \begin{cases} \log p, & n = p^k \text{ is a power of a prime } p \\ 0 & \text{otherwise} \end{cases}$$

Show that for  $s > 1$ ,

$$-\frac{\zeta'(s)}{\zeta(s)} = \sum_{n=1}^{\infty} \frac{\Lambda(n)}{n^s}$$

4. Compute the character tables of  $(\mathbb{Z}/q\mathbb{Z})^*$  for  $q = 8, 15$ .