

**ADVANCED NUMBER THEORY 2008
ASSIGNMENT 3**

Due date: Wednesday, June 18, 2008

1. A binary quadratic form $ax^2 + bxy + cy^2$ is *primitive* if $\gcd(a, b, c) = 1$. For $d = 0, 1 \pmod{4}$ we set $h(d)$ to be the number of $SL_2(\mathbb{Z})$ -equivalence classes of primitive forms of discriminant d (in the case of negative discriminant $d < 0$, we consider only *positive definite* forms, equivalently those with $a > 0$). Let $H(d)$ to be the number of classes of *all* forms of discriminant d (positive definite for $d < 0$). Show that

$$H(d) = \sum_{g^2|d} h\left(\frac{d}{g^2}\right)$$

2. Find the class number $h(d)$ and reduced representatives of the equivalence classes for all discriminants $-24 \leq d \leq -4$.

Note: This is completely mechanical with the reduction theory that we learnt in class. If you feel like it, write a computer program to do this, in which case you may list all class numbers for the first 50 negative discriminants; what do we learn from these?