ADVANCED NUMBER THEORY 2008
ASSIGNMENT 4

Due date: Wednesday, June 25, 2008

1. Decide which of the following forms are equivalent:

$$[6, 12, 7], \quad [3, 6, 5], \quad [5, 14, 11] .$$

2. An *automorph* of a binary quadratic $f = [a, b, c]$ is a unimodular transformation $\begin{pmatrix} m & n \\ k & l \end{pmatrix} \in SL_2(\mathbb{Z})$ which transforms f to itself. If f is anisotropic ($\mathrm{disc}(f)$ is not a perfect square) and primitive ($\gcd(a, b, c) = 1$), then all automorphs are of the form

$$P(t, u) = \begin{pmatrix} \frac{t-bu}{2} & -cu \\ au & \frac{t+bu}{2} \end{pmatrix}$$

where (t, u) solves the Pell equation $t^2 - Du^2 = 4$, $D = \mathrm{disc}(f)$.

When the discriminant $D > 0$ is not a perfect square, the group of automorphs is infinite, of the form $\pm P_0^n$, $n \in \mathbb{Z}$ where $P_0 = P(t_0, u_0)$, with $t_0 > 0$, $u_0 > 0$ is the minimal solution of the Pell equation $t^2 - Du^2 = 4$.

Find the generator P_0 for the forms $[3, 1, -1]$, $[-3, 3, 1]$, $[1, 0, -58]$, $[2, 0, -29]$.

3. a) Find all automorphs of the isotropic form $f(x, y) = xy$.
*b) Do the same for all isotropic nondegenerate ($\mathrm{disc}(f) \neq 0$) forms.
(*=extra credit).