

**ADVANCED NUMBER THEORY 2008
ASSIGNMENT 5**

Due date: Wednesday, July 2, 2008

1. Find which primes p can be written in the form $p = x^2 + 5xy + 11y^2$ and find the total number of all integer solutions for this equation.
2. Show that an integer $n > 1$ is a sum of two squares if and only if in the prime decomposition of n , any prime $q = 3 \pmod{4}$ has to appear with an even power, that is

$$n = 2^\alpha \prod_{p=1 \pmod{4}} p^{\beta(p)} \prod_{q=3 \pmod{4}} q^{2\gamma(q)}$$

In that case, show that the total number of ways of writing n as a sum of two squares is $4 \prod_p (\beta(p) + 1)$.

3. Show that if $D = 1 - 4q$, $q > 1$, and $h(D) = 1$ then q is prime.
4. Show that if $D = -4q$, $q > 1$, and $h(D) = 1$ then q is a prime power.