

Advanced Number Theory 2013

Assignment 1

Due date: March 12, 2013

1. Compute $\pi_q(18)$, the number of monic irreducible polynomials of degree 18 in $F_q[x]$.

2. Show that
$$\sum_{\substack{N \text{ monic} \\ \deg N < x}} \frac{1}{|N|} = x$$

3. The von Mangoldt function for $F_q[x]$ is defined as $\Lambda(f) = \deg P$ if $f = cP^k$ with P prime and $c \in F_q^x$ a scalar, and $\Lambda(f) = 0$ otherwise. Compute the sum

$$\sum_{\substack{\deg f < x \\ f \text{ monic}}} \frac{\Lambda(f)}{|f|}$$

4. Use the Prime Polynomial Theorem to show that

$$\sum_{\substack{P \text{ prime} \\ \deg P < x}} \frac{1}{|P|} \sim \log x$$

as $x \rightarrow \infty$.