TOPICS IN NUMBER THEORY 2013
ASSIGNMENT 3
DUE DATE: APRIL 30, 2013

1. Show that given distinct elements $\alpha_1, \ldots, \alpha_r \in \mathbb{F}_q$, and nonzero $\beta_1, \ldots, \beta_r \in \mathbb{F}_q^\times$, there are infinitely many monic irreducible polynomials $P \in \mathbb{F}_q[x]$ for which $P(\alpha_j) = \beta_j$, $j = 1, \ldots, r$. (Hint: First try the case $P(0) = 1$).

2. Let $Q \in \mathbb{F}_q[x]$ be a polynomial of positive degree, and χ a Dirichlet character modulo Q. We said that χ is *even* if $\chi(cf) = \chi(f)$ for all units $c \in \mathbb{F}_q^\times$. Show that the number of even characters modulo Q is $\Phi(Q)/(q-1)$.

3. Show that if $\chi \neq \chi_0$ is nontrivial and even, then the L-function $L(s, \chi)$ has a "trivial zero" at $s = 0$, so that $L(s, \chi) = (1-u)P(u)$, $u = q^{-s}$, with $P(u)$ a polynomial of degree $\leq \deg Q - 2$.