Exercise 1. For an odd prime $p$, let $g(p) > 1$ be the least primitive root modulo $p$. Show that $\lim \sup_p g(p) = \infty$.

Exercise 2. Let $p \neq q$ be distinct odd primes, and set $N := p \cdot q$. Show that there is an integer $a$ coprime to $N$ for which $a^{(N-1)/2} \neq \left(\frac{a}{N}\right)$ mod $N$, where $\left(\frac{a}{N}\right)$ is the Jacobi symbol.

Explain why we expect to find such $a$ which is $O((\log N)^2)$.

Exercise 3. The Gauss sum associated to a Dirichlet character $\chi \bmod p$ is

$$\tau(\chi) := \sum_{a \bmod p} \chi(a)e^{2\pi i \frac{a}{p}}$$

For $p \neq 2$ prime, show that for the quadratic character $\chi_p(t) = \left(\frac{t}{p}\right)$ (Legendre symbol),

$$\tau(\chi_p)^2 = \left(\frac{-1}{p}\right) p.$$