NUMBER THEORY SEMINAR 2014/15
EXERCISE 2
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1. Let $d_n = \mathrm{lcm}(1, 2, \ldots, n)$. Use the Prime Number Theorem (in the form $\psi(x) \sim x$) to show that $\log d_n \sim n$ as $n \to \infty$.

2. A real number x has an irrationality measure μ if and only if $\forall \epsilon > 0$, $\exists Q > 1$ so that for $q > Q$, $|x - \frac{p}{q}| > 1/q^{\mu+\epsilon}$.

Assume that we know the $\mu(2^{1/3}) < 3$ (this was proved by Axel Thue). Show that the equation $x^3 - 2y^3 = 1$ has only finitely many integer solutions.

3. Euler showed that the continued fraction expansion of $\mathbf{e}$ is

$$\mathbf{e} = [2; 1, 2, 1, 1, 4, 1, 1, 6, \ldots, 1, 1, 2n, \ldots]$$

Use this to show that the irrationality measure of $\mathbf{e}$ is $\mu = 2$.