NUMBER THEORY SEMINAR 2014/15
EXERCISE 3
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1. Show that there is some $c > 0$ so that for all $x \gg 1$, every interval $(x - cx^{1/4}, x)$ contains a sum of two squares.
Hint: Given $y \gg 1$, show that there is a square $y - O(\sqrt{y}) < \square < y$.
It is not known if we may take c arbitrarily small.

2. Show that there is some $c > 0$ for which there are arbitrarily large x for which there is an interval $[x, x + c\frac{\log x}{\log\log x}]$ which contains no sums of two squares.
Hint: If p_j is the j-th prime congruent to 3 mod 4, find n with $n + j = p_j \bmod p_j^2$, $j = 1, \dots, J$.