1. Let $k$ be an even integer, and $p$ a prime so that $2p - 1$ is also prime, and such that both $p$ and $2p - 1$ are coprime to $k$. Let $n = k(2p - 1)$. Show that $\varphi(n) = \varphi(n + k)$ (where $\varphi$ is Euler totient function).

2. Show that the probability that a random permutation on $n$ letters is an $n$-cycle, is $1/n$.

3. Let $\Omega_n(\sigma)$ be the number of cycles of a permutation $\sigma \in S_n$, and let

$$f_n(t) := E(e^{it\Omega_n})$$

be the characteristic function of $\Omega_n$, thought of as a random variable on $S_n$. Here $E$ denotes the expectation, that is the average over all permutations in $S_n$. Show that

$$f_n(t) = \prod_{j=1}^{n}(1 - \frac{1}{j} + \frac{e^{it}}{j}).$$