EXERCISE 1
NUMBER THEORY RESEARCH SEMINAR 2017/18
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1. Let k be an even integer, and p a prime so that $2p-1$ is also prime, and such that both p and $2p-1$ are coprime to k. Let $n = k(2p-1)$. Show that $\varphi(n) = \varphi(n+k)$ (where φ is Euler totient function).

2. Show that the probability that a random permutation on n letters is an n-cycle, is $1/n$.

3. Let $\Omega_n(\sigma)$ be the number of cycles of a permutation $\sigma \in S_n$, and let

$$f_n(t) := \mathrm{E}(e^{it\Omega_n})$$

be the characteristic function of Ω_n, thought of as a random variable on S_n. Here E denotes the expectation, that is the average over all permutations in S_n. Show that

$$f_n(t) = \prod_{j=1}^{n} (1 - \frac{1}{j} + \frac{e^{it}}{j}) \ .$$