Exercise 1. Let $k \geq 12$. Show that the set of Poincaré series
$$\left\{ p_k^m, \ 1 \leq m \leq \frac{k}{12} + 1 \right\}$$
span the space of cusp forms $S_k$.

Exercise 2. Let $c \geq 1$ and $m, n \in \mathbb{Z}$ integers. The Kloosterman sum is defined as
$$\text{Kl}(m, n; c) := \sum_{x \mod c, \gcd(x, c) = 1} e\left(\frac{mx + n\bar{x}}{c}\right)$$
where we abbreviate
$$e(z) := e^{2\pi iz}$$
and $\bar{x}$ denotes the multiplicative inverse of $x$ mod $c$: $x\bar{x} = 1 \mod c$.

a) Show that if $p$ is prime and $a \neq 0 \mod p$ then $\text{Kl}(a, 0; p) = -1$.

b) If $a, b \neq 0 \mod p$ then
$$\text{Kl}(a, b; p) = \text{Kl}(ab, 1; p)$$

Exercise 3. a) Show that the hyperbolic measure $\frac{dx dy}{y^2}$ is invariant under an Moebius transformation $g \in \text{SL}(2, \mathbb{R})$.

b) Compute the hyperbolic area $\int_{\mathcal{F}} \frac{dx dy}{y^2}$ of the standard fundamental domain $\mathcal{F}$ for $\text{SL}(2, \mathbb{Z})$. 
