Number Theory Homework #1

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To be handed in on Monday, November 7, 2011.

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1. Show that the squares of integers are congruent to either 0 or 1 modulo 4: $x^2=0,1 \bmod 4$.

2. Show that an integer which is congruent to 3 modulo 4 is not a sum of two squares: $n \neq x^2+y^2$.

3. A nilpotent element in $\mathbb{Z}/N\mathbb{Z}$ is a residue $m \neq 0$ for which there is some $r>1$ with $m^r=0 \bmod N$. Show that if N is not square-free then there are nilpotents in $\mathbb{Z}/N\mathbb{Z}$. (We say N is square-free if it is not divisible by a^2 for any integer $a>1$).

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Course homepage: http://www.math.tau.ac.il/~rudnick/courses/int_numth.html