

Number Theory Homework #10

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To be handed in by Monday, January 16, 2012.

1. Show that the logarithmic integral $\text{Li}(x)$ admits an asymptotic expansion of the form:

$$\mathbf{Li}(x) = \frac{x}{\log x} + \frac{x}{(\log x)^2} + O\left(\frac{x}{(\log x)^3}\right)$$

2. A form of the Prime Number Theorem (PNT) states that

$$\pi(x) = \frac{x}{\log x} + O\left(\frac{x}{(\log x)^2}\right), \quad \text{as } x \rightarrow \infty$$

Show that PNT implies “Bertand’s Postulate”: For x sufficiently large, there is a prime between x and $2x$.

3. The von Mangoldt function $\Lambda(n)$ is defined as $\Lambda(p^k) = \log p$ for p prime and $k > 0$, and $\Lambda(n) = 0$ if n is not a prime power (Λ is the Greek letter Lambda). Show that $\sum_{d|n} \Lambda(d) = \log n$, the sum over all divisors d of n .
4. Suppose that $N = pq$ is a product of two distinct primes. Show that if we know both N and $\varphi(N)$ then we can quickly recover the factors p and q .

Course homepage: http://www.math.tau.ac.il/~rudnick/courses/int_numth.html