Number Theory Homework #11

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To be handed in by Monday, January 23, 2012.

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1. For the following pairs a, b of Gaussian integers, find Gaussian integers k, r with $a=kb+r$ and $N(r) < N(b)$:

   i) $a=7$, $b = 2-\mathbf{i}$
   ii) $a=5+\mathbf{i}$, $b = 3+3\mathbf{i}$
   iii) $a=14+5\,\mathbf{i}$, $b = 5+3\mathbf{i}$

2. Find $\gcd(a, b)$ for the pairs of Gaussian integers in problem 2.

3. Find the factorization into irreducibles in the ring of Gaussian integers, of

   $14 + 5\,\mathbf{i}$, $5 + \mathbf{i}$, $70+35\,\mathbf{i}$,

4. Find all integer solutions of the equation $x^2 - y^2=1$.

Course homepage: http://www.math.tau.ac.il/~rudnick/courses/int_numth.html