Number Theory Homework #3

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To be handed in by Monday, November 21, 2011.

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1. Given integers $a_0, a_1, \ldots, a_N$ with $a_i > 0$ for $i > 0$, we denote by $[a_0; a_1, \ldots, a_N]$ the continued fraction

$$[a_0; a_1, \ldots, a_N] = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_{2+\ddots \frac{1}{a_N}}}}$$

   Define $p_0 = a_0$, $q_0 = 1$, $p_1 = a_0 a_1 + 1$, $q_1 = a_1$, and by recursion:

   $p_n = a_n p_{n-1} + p_{n-2}$, $q_n = a_n q_{n-1} + q_{n-2}$.

   Show that $p_n q_{n-2} - p_{n-2} q_n = (-1)^n a_n$.

2. a) Compute the continued fraction expansion $\sqrt{5}$ and of $\sqrt{7}$.

   b) What number has the continued fraction expansion [1;2,2,2,2,.....] ?

3. Find the inverses mod 41 of all residues x=11,12,...,20 mod 41.

4. Compute Euler's φ-function for $121 \leqq n \leqq 130$.

Course homepage: http://www.math.tau.ac.il/~rudnick/courses/int_numth.html