Number Theory Homework #4

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To be handed in by Monday, November 28, 2011.

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1. Find the solutions of the following systems of congruences

    a) $x = a$ mod 17, $x = b$ mod 8.

    b) $x = a$ mod 17, $x = b$ mod 7.

    c) $x = A$ mod 3, $x = B$ mod 5, $x = C$ mod 7

2. Find the greatest common divisor in the ring $\mathbb{Z}/p\mathbb{Z}[x]$ of the polynomials

    $f(x) = x^3 - 2x^2 + x - 2$, $g(x) = x^4 - 2x^3 + 7x - 14$, for all primes $p = 2, \ldots, 19$.

3. Find a decomposition into irreducibles in the ring $\mathbb{Z}/p\mathbb{Z}[x]$ of the polynomials : $x^2 + 1$, $x^2 - 2$, $x^3 - 2$, $x^3 + 3x^2 + 2x + 6$, for $p = 5$ and $p = 7$.

4. Show that there are infinitely many primes of the form 4k+3.