# Number Theory Homework #8

## Prof. Zeev Rudnick

To be handed in by Monday, December 26, 2011.

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1. Compute the following Legendre symbols:

   a) $\left(\frac{3}{29}\right), \left(\frac{-14}{73}\right), \left(\frac{22}{103}\right), \left(\frac{501}{1223}\right), \left(\frac{365}{1847}\right)$.

   b) $\left(\frac{n}{61}\right)$ for $2 \le n \le 10$.

2. Find all primes *p* for which the following congruences are solvable:

   i) $x^2=13 \bmod p$ ii) $x^2=14 \bmod p$ iii) $x^2 + 4x + 6 = 0 \bmod p$

3. Compute the following Jacobi symbols:

$$\left(\frac{5}{91}\right), \left(\frac{-18}{91}\right), \left(\frac{80}{91}\right), \left(\frac{75}{77}\right), \left(\frac{24}{77}\right), \left(\frac{58}{77}\right).$$

4. Decide which of the following congruences are solvable, and if so, find all solutions:

   a) $x^2=a \bmod 91$, a=5, -18, -17. b) $x^2=b \bmod 105$, b=1,-19, 46.

5. a) Let p be a prime of the form 4q+1 where q is also a prime. Show that 2 is a primitive root modulo p.

   b) Find 5 examples of such primes.

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