Number Theory Homework #1

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To be handed in on Monday, November 14, 2016.

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1. **i)** Show that the squares of integers are congruent to either 0 or 1 modulo 4: $x^2=0,1 \bmod 4$.

   ii) Show that an integer which is congruent to 3 modulo 4 is not a sum of two squares: $n \neq x^2+y^2$.

2. **i)** Show that an integer is divisible by 9 if and only if the sum of its digits is divisible by 9.

   ii) Show that an integer is divisible by 11 if and only if the alternating sum of its digits is divisible by 11 (If $A=a_0 + a_1 10^1 + a_2 10^2 + \ldots$, with $0 \le a_i \le 9$ then the alternating sum of digits is $a_0 - a_1 + a_2 - \ldots$).

3. Give the continued fraction expansion of the rationals 17/6, 53/22, 357/52.

4. Find the rational numbers represented by the continued fractions [3;5,8], [1;2,3,4], [1;1,1,1], [1,1,1,1,1].

5. Find all integer solutions to the equations $24x + 138y = 18$, $56x + 72y = 40$.

6. The Fibonacci numbers are defined by the recursion $F_{n+2} = F_{n+1} + F_n$, with initial conditions $F_0=0$, $F_1=1$. Show that the number of steps in the Euclidean algorithm (with positive remainder) to compute GCD(F_{n+1}, F_n) is at least $C \log_2 F_n$, for *n* sufficiently large and a suitable constant C>0.

   Hint: Show that the number of step is n-3, and that $\boldsymbol{F_n} = \frac{\gamma_+^{\boldsymbol{n}} - \gamma_-^{\boldsymbol{n}}}{\sqrt{5}}$, $\gamma_\pm = \frac{1 \pm \sqrt{5}}{2}$.

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Course homepage: http://www.math.tau.ac.il/~rudnick/courses/int_numth.html