Number Theory Homework #10

Prof. Zeev Rudnick

To be handed in on Monday, January 16, 2017.

WARNING: SAVE A COPY OF THE HOMEWORK ASSIGNMENT IN CASE YOU DON'T HAVE TIME TO COLLECT IT !

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1. Show that if an integer is a sum of two squares: $n=x^2+y^2$, then in the prime power decomposition of n, all primes p=3 mod 4 appear with **even** exponents: for p=3 mod 4, if $p^k \mid n$ but $p^{k+1} \nmid n$ then k is even.

2. Show that the ring $\mathbb{Z}[\sqrt{-6}] = \{\boldsymbol{m} + \boldsymbol{n}\sqrt{-\boldsymbol{6}} : \boldsymbol{m}, \boldsymbol{n} \in \mathbb{Z}\}$ does **not** have unique factorization into irreducibles.

3. Show that $\mathbb{Z}[\sqrt{-2}] = \{\boldsymbol{m} + \boldsymbol{n}\sqrt{-\boldsymbol{2}} : \boldsymbol{m}, \boldsymbol{n} \in \mathbb{Z}\}$ is a Euclidean ring with respect to the standard complex norm $N(z) = z \cdot \bar{z}$.

4. Show that if $n = 5{,}7 \bmod 8$ then it is not of the form $x^2 + 2y^2$. (Here n, x,y are integers).

5. Show that an odd prime p is of the form $p = x^2 + 2y^2$ if and only if p=1,3 mod 8.

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Course homepage: http://www.math.tau.ac.il/~rudnick/courses/int_numth.html