Number Theory Homework #3

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To be handed in on Monday, November 28, 2016.

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1. Given integers $a_0; a_1, \dots, a_N$ with $a_i \geq 1$ for i>0, we denote by $[a_0; a_1, \dots, a_N]$ the continued fraction

$$[a_0; a_1, \dots, a_N] = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_{2+\ddots\frac{1}{a_N}}}}$$

Define $p_{-1} = 1$, $q_{-1}=0$, $p_0 = a_0$, $q_0=1$, and by recursion:

$p_n = a_n p_{n-1} + p_{n-2}$, $q_n = a_n q_{n-1} + q_{n-2}$.

Show that i) $p_n q_{n-1} - p_{n-1} q_n = (-1)^{n-1}$

ii) $p_n q_{n-2} - p_{n-2} q_n = (-1)^n a_n$

2. Find the greatest common divisor of the polynomials

   $f(x)= x^3-2x^2+x-2$, $g(x) = x^4 -2x^3 +7x-14$ in $\mathbf{Z}/p\mathbf{Z}[\mathbf{x}]$, for all primes p up to 19.

3. i) Find the decomposition into monic irreducibles of the following polynomials in $\mathbf{Z}/5\mathbf{Z}[x]$:

   $x^2 + 1$, $x^2 - 2$, $x^3 - 2$, $x^3 + 3x^2 + 2x + 6$.

   ii) Do the same in $\mathbf{Z}/7\mathbf{Z}[x]$.

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Course homepage: http://www.math.tau.ac.il/~rudnick/courses/int_numth.html