Number Theory Homework #7

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To be handed in on Monday, December 26, 2016.

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1. Compute the following Legendre symbols:

   a) $\left(\frac{3}{29}\right), \left(\frac{-14}{73}\right), \left(\frac{22}{103}\right), \left(\frac{501}{1223}\right), \left(\frac{365}{1847}\right).$

   b) $\left(\frac{n}{61}\right)$ for $2 \le n \le 10$.

2. Find all primes p for which the following congruences are solvable:

   i) $x^2 = 13 \bmod p$ ii) $x^2 = 14 \bmod p$ iii) $x^2 + 4x + 6 = 0 \bmod p$

3. Compute the following Jacobi symbols:

$$\left(\frac{5}{91}\right), \left(\frac{-18}{91}\right), \left(\frac{80}{91}\right), \left(\frac{75}{77}\right), \left(\frac{24}{77}\right), \left(\frac{58}{77}\right).$$

4. Decide which of the following congruences are solvable, and if so, find all solutions:

   a) $x^2 = a \bmod 91$, $a = 5, -18, -17$. b) $x^2 = b \bmod 105$, $b = 1, -19, 46$.

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Course homepage: http://www.math.tau.ac.il/~rudnick/courses/int_numth.html