

# Number Theory Homework #9

Prof. Zeev Rudnick

To be handed in on Monday, January 9, 2017.

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1. A form of the Prime Number Theorem (PNT) states that

$$\pi(x) = \text{Li}(x) + O\left(\frac{x}{(\log x)^2}\right), \quad \text{as } x \rightarrow \infty$$

where  $\text{Li}(x) := \int_2^x \frac{dt}{\log t}$  is the logarithmic integral. Show that the PNT implies “Bertrand’s Postulate”: For  $x$  sufficiently large, there is a prime between  $x$  and  $2x$ .

2. For the following pairs  $a, b$  of Gaussian integers, find Gaussian integers  $k, r$  with  $a = kb + r$  and  $N(r) < N(b)$ :

- i)  $a = 7, b = 2 - \mathbf{i}$
- ii)  $a = 5 + \mathbf{i}, b = 3 + 3\mathbf{i}$
- iii)  $a = 14 + 5\mathbf{i}, b = 5 + 3\mathbf{i}$

3. Find  $\gcd(a, b)$  for the pairs of Gaussian integers in problem 2.

4. Find the factorization into irreducibles in the ring of Gaussian integers, of

$$14 + 5\mathbf{i}, \quad 5 + \mathbf{i}, \quad 70 + 35\mathbf{i},$$

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Course homepage: [http://www.math.tau.ac.il/~rudnick/courses/int\\_numth.html](http://www.math.tau.ac.il/~rudnick/courses/int_numth.html)