

EXERCISE 5
OPEN PROBLEMS IN NUMBER THEORY 2017/18
DUE DATE: MAY 30, 2018

Exercise 1. *Show that we can have unbounded number of lattice points in very small caps on the sphere $x^2 + y^2 + z^2 = R^2$ in 3 dimensions.*

Hint: Given $K > 1$, find an integer n which has $r_2(n) \geq K$, and take $R^2 = N^2 + n$.

Exercise 2. *Show that there is some $c > 0$ so that all lattice points in a cap of diameter $cR^{1/4}$ on the sphere $x^2 + y^2 + z^2 = R^2$ are co-planar.*