Number Theory Seminar - Exercise 4

1. Use the explicit formula to derive that $\pi(n) \leq \frac{q^n}{n}$.

2. Calculate the value of $\pi(18)$ over $\mathbb{F}_q[x]$.

3. Define the von-Mangoldt function, for monic polynomials f:
$\Lambda(f) = \begin{cases} 0 & f \text{ is not a prime power} \\ deg(P) & f = P^k \text{ is a power of a prime (i.e. monic irreducible)} \end{cases}$
Show that $\sum_{\deg f = n} \Lambda(f) = q^n$ the sum over all monic polynomials of degree n.

4. a) Show that $\sum_{\substack{\text{N monic} \\ \deg(N) < x}} \frac{1}{|N|} = x$.

b) Use the PPT to show that $\sum_{\substack{\text{p monic} \\ \deg(p) < x}} \frac{1}{|P|}$ is asymptotically $\log(x)$.