

**Number Theory Seminar**  
**Assignment #5**

1. Compute  $\left(\frac{x^2-1}{x^3+2x+1}\right)_2$

[Hint: In class we have solved  $(x^2 - 1)^2$ .

Find  $(x^2 - 1)^4, (x^2 - 1)^8$ .

Then you get  $(x^2 - 1)(x^2 - 1)^4(x^2 - 1)^8 = (x^2 - 1)^{13}$ ]

2. Compute  $\left(\frac{x+1}{x^3+2x+1}\right)_2$  using the reciprocity law.

3. Given a prime polynomial  $P$  check when  $\left(\frac{a}{P}\right)_2 = 1$ , if  $a = x$

In  $\mathbb{F}_7[x]$

In  $\mathbb{F}_{13}[x]$

4. Over  $\mathbb{F}_5[x]$ , let  $P$  be a monic irreducible polynomial (prime). Prove that  $(x^2 - 1) \equiv y^2 \pmod{P}$  is solvable for  $P$  such that  $P(\pm 1) \neq 0$  iff  $P(1)P(-1) = \pm 1$