Exercise 1. Show that the sequence
\[
\{0, 0, 1, 0, 1, 2, 0, 1, 2, 3, 0, \ldots\}
\]
is uniformly distributed modulo one.
Hint: Use Weyl’s criterion.

Exercise 2. Let \(\varphi = (1 + \sqrt{5})/2\) be the golden mean. Show that the sequence \(\{\varphi^n : n = 1, 2, 3, \ldots\}\) is not dense modulo one, in fact that the only limit points are 0 and 1.
Hint: Let \(\tilde{\varphi} = (1 - \sqrt{5})/2\). Show \(U_n := \varphi^n + (\tilde{\varphi})^n\) is an integer, and hence \(\text{dist}(\varphi^n, \mathbb{Z}) \to 0\) as \(n \to \infty\).

Exercise 3. Show that as \(N \to \infty\),
\[
\sum_{n=1}^{N} \frac{\log n}{n} = \frac{1}{2} (\log N)^2 + O(1).
\]
Hint: Use Euler’s summation formula.