SOME NEW PERSPECTIVES IN BEST APPROXIMATION AND INTERPOLATION OF RANDOM DATA

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A new notion of universally optimal experimental design is introduced, relevant from the perspective of adaptive nonparametric estimation. It is demonstrated that both discrete and continuous Chebyshev designs are universally optimal in the problem of fitting properly weighted algebraic polynomials to random data.

Optimal interpolating designs in rational regression proved particularly elusive in the past. The question can be handled using its connection to elliptic interpolation, in which the ordinary circular sinus, appearing in the classical trigonometric interpolation, is replaced by the Abel-Jacobi elliptic sinus $\operatorname{sn}(x, k)$ of a modulus k.

First, it will be demonstrated that – in a natural setting of equidistant design – the elliptic interpolant is never optimal in the so-called normal case $k \in (-1, 1)$, except for the trigonometric case k = 0. However, the equidistant elliptic interpolation is always optimal in the imaginary case $k \in i\mathbb{R}$.

Through a relation between elliptic and rational functions, the result leads to a long sought optimal design, for properly weighted rational interpolants. Both the poles and nodes of the interpolants can be conveniently expressed in terms of classical Jacobi's theta functions.