Space-Time Covariance Functions for Planet Earth

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In this paper, we propose stationary covariance functions for processes that evolve temporally over a sphere, as well as cross-covariance functions for multivariate random fields defined over a sphere. For such processes, the great circle distance is the natural metric that should be used in order to describe spatial dependence. Given the mathematical difficulties for the construction of covariance functions for processes defined over spheres cross time, approximations of the state of nature have been proposed in the literature by using the Euclidean (based on map projections) and the chordal distances. We present several methods of construction based on the great circle distance and provide closed-form expressions for both spatio-temporal and multivariate cases. A simulation study assesses the discrepancy between the great circle distance, chordal distance and Euclidean distance based on a map projection both in terms of estimation and prediction in a space-time and a bivariate spatial setting, where the space is in this case the Earth. We revisit the analysis of Total Ozone Mapping Spectrometer (TOMS) data and investigate differences in terms of estimation and prediction between the aforementioned distance-based approaches. Both simulation and real data highlight sensible differences in terms of estimation of the spatial scale parameter. As far as prediction is concerned, the differences can be appreciated only when the interpoint distances are large, as demonstrated by an illustrative example.

Key words: Chordal distance; Cokriging; Global data; Great circle distance; Multivariate; Spatio-temporal statistics; Sphere.