Charms, Charrelations and their Relations to Correlations

Classical Higher-Order Statistics (HOS, usually in the form of high-order cumulants) are a powerful tool in the context of multivariate statistical analysis, often entailing valuable statistical information beyond Second-Order statistics (SOS), albeit at the expense of increased computational and notational complexity and compromised statistical stability (in the sense that longer observation intervals might be required in order to fully realize the advantages of HOS over SOS).

In this talk we introduce alternative generic tools, offering the structural simplicity and controllable statistical stability of SOS on the one hand, yet retaining higher-order statistical information on the other hand. While cumulants are related to high-order derivatives of the log characteristic function at the origin, our new tools are related to lower-order (first and second) derivatives away from the origin, at locations called "*processing-points*", and are termed "*charmean*" (or "*charm*", in short) and "*charrelation*". The charm and charrelation coincide with the classical mean and covariance (resp.) when the *processing-point* approaches the origin, but can offer continuously tunable tradeoffs between statistical stability and information contents as the *processing-point* is dragged away from the origin. The use of *charm* and *charrelation* as HOS-substitutes to the mean and correlation can therefore lead to considerable performance improvement in some classical estimation problems in signal processing, such as Blind Source Separation or estimation of the parameters of a (non-Gaussian) Auto-Regressive (AR) process.

In addition to introducing the underlying mathematical-statistical concepts for the development and analysis of charm- and charrelation-based estimation, we provide expressions for the asymptotic bias and variance of their sample-estimates. The availability of such expressions enables data-driven selection of the *processing-points*, so as to minimize the predicted mean square estimation error in a given problem – as we demonstrate for the above-mentioned signal-processing related examples.