2SAT∨

¬2SAT

On Variants of SAT Problems

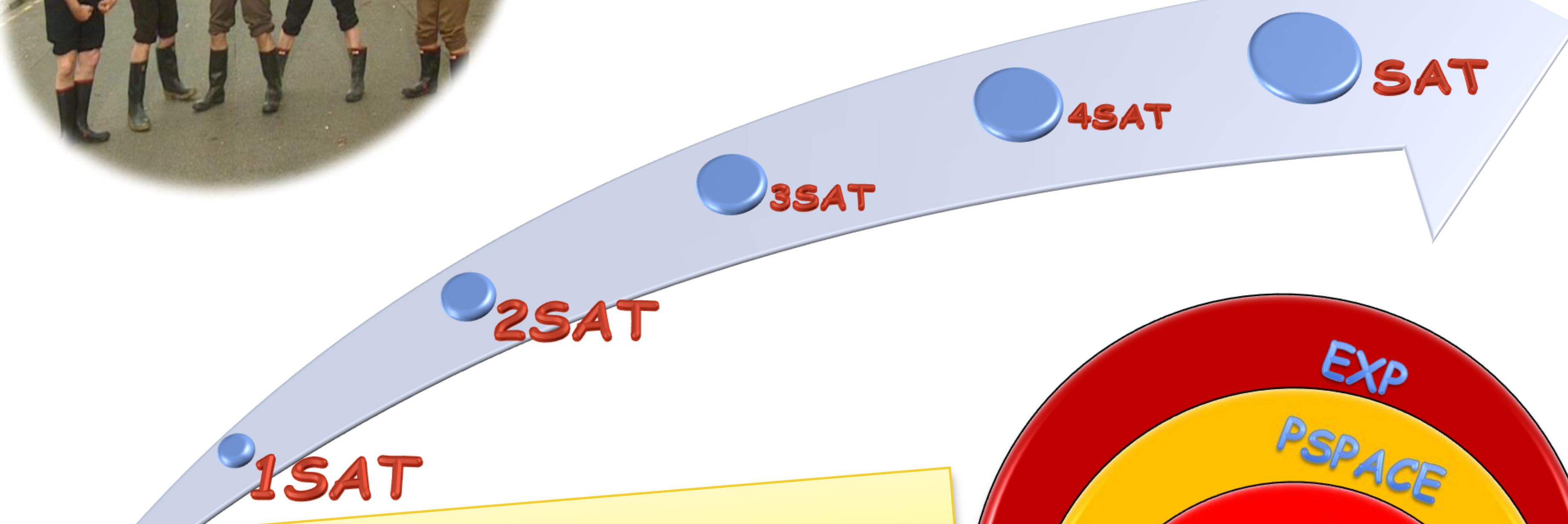
Goal:

- Discuss the complexity of variants of SAT

Plan:

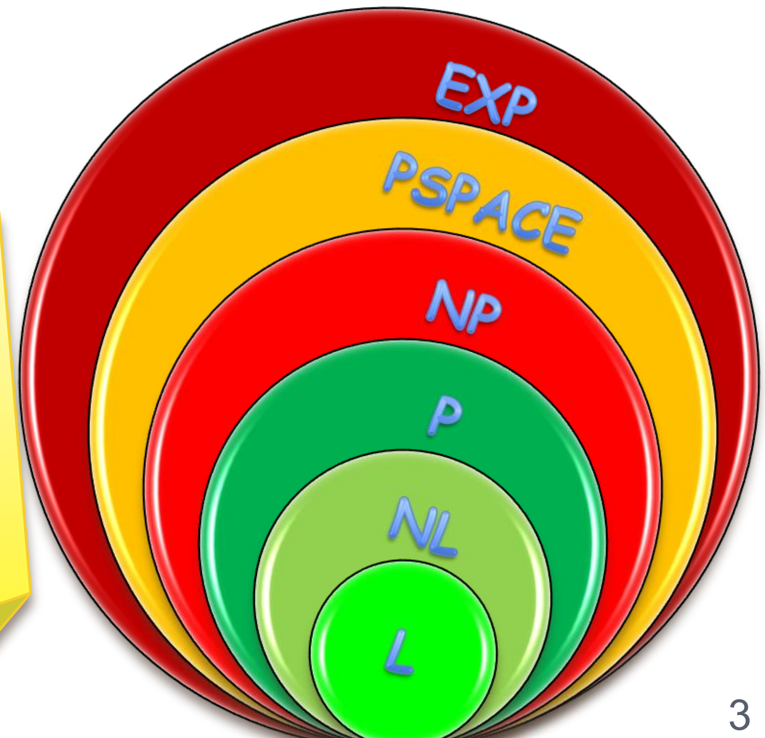
- General
- 2SAT
- Max2SAT

Special cases of SAT



2SAT

- In P?
- NL-complete?
- Variants?



2SAT

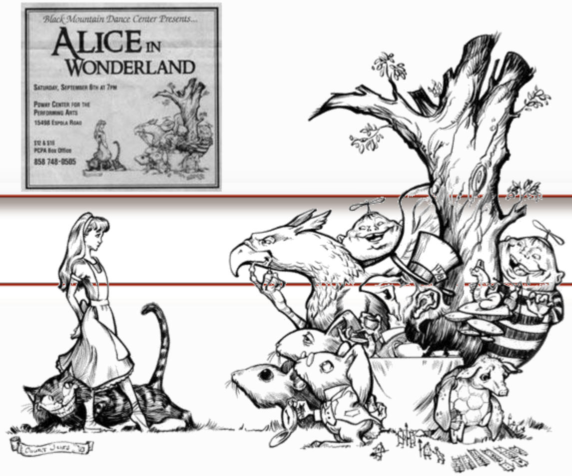
2SAT Instance:

- a 2-CNF formula φ

EG $(\neg x \vee y) \wedge (\neg y \vee z) \wedge (x \vee \neg z)$

Decision Problem:

- is φ satisfiable?



Theorem:

- 2SAT $\in$ P

Proof:

- Reduce 2SAT to a graph problem in P: construct G_φ -- then specify problem

Implication graph $G_\varphi=(V_\varphi, E_\varphi)$

V_φ

- 1 vertex for every literal of φ

E_φ

- edge (α,β) $\Leftrightarrow$ φ contains clause $(\neg\alpha\vee\beta)$

note
edges: $(\alpha,\beta)\in E_\varphi \Leftrightarrow (\neg\beta,\neg\alpha)\in E_\varphi$
paths: $\alpha \mapsto \beta \Leftrightarrow \neg\beta \mapsto \neg\alpha$

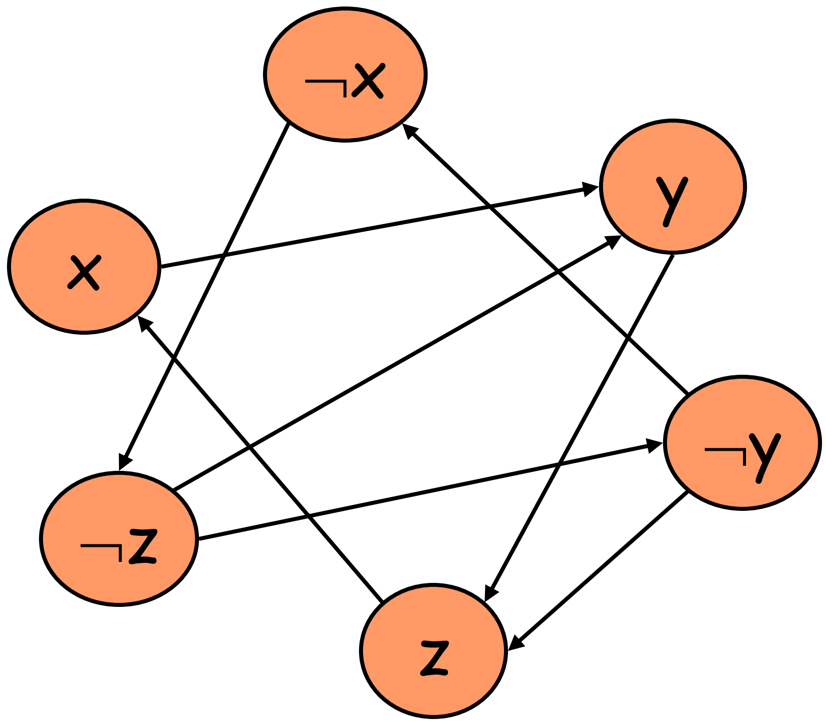
Theorem:

- φ is unsatisfiable $\Leftrightarrow$
$\exists x$ s.t. $x \mapsto \neg x$ and $\neg x \mapsto x$ in G_φ

note
$\alpha \mapsto \beta \Rightarrow$
$\alpha \Rightarrow \beta$

Implication graph : Example

$(\neg x \vee y) \wedge (\neg y \vee z) \wedge (x \vee \neg z) \wedge (z \vee y)$



Com

Correctness

Completeness:

- $x \mapsto \neg x \Rightarrow$ can't assign TRUE to x
- $\neg x \mapsto x \Rightarrow$ can't assign FALSE to x

Soundness:

- Repeat
  Pick an x; if $x \mapsto \neg x$, $\alpha=\neg x$ o/w $\alpha=x$ -
  no $\alpha \mapsto \neg\alpha$, hence assign TRUE to α;
  Then, $\forall$ literal β s.t. $\alpha \mapsto \beta$:
  assign TRUE to β and FALSE to $\neg\beta$
- No inconsistencies!

note

$\alpha \mapsto \beta \wedge \alpha \mapsto \neg\beta$
$\Rightarrow \alpha \mapsto \neg\alpha$

Graph Connectivity (CONN)

CONN Instance:

- a directed graph $G=(V,E)$ and 2 vertices $s,t \in V$

Decision Problem:

- Is there is a path from s to t in G?

Theorem:

- CONN $\in$ P

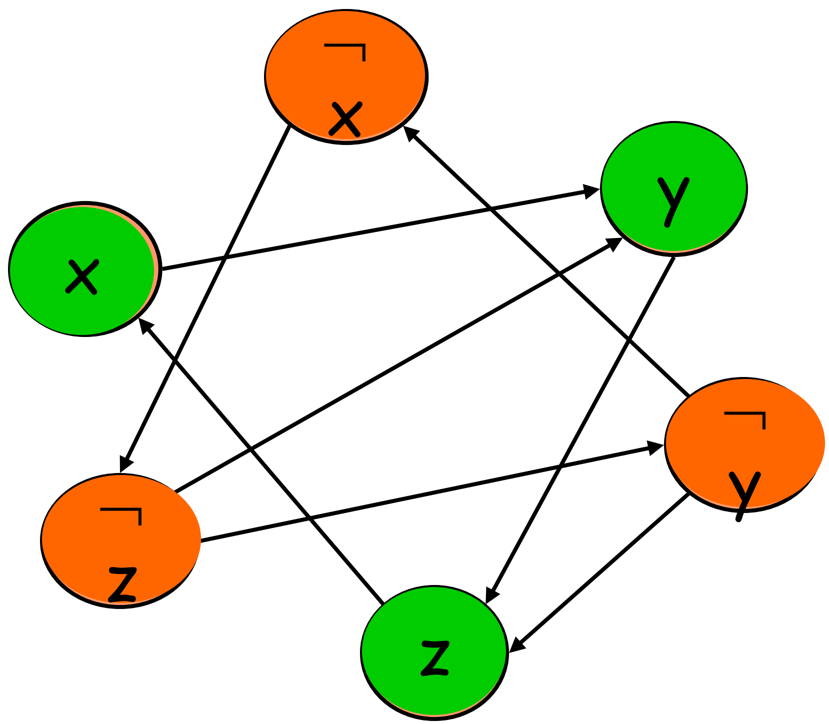
Apply some search algorithm (DFS/BFS).

Corollary:

- "$\exists x$ s.t. $x \mapsto \neg x$ and $\neg x \mapsto x$ in G_φ" $\in$ P ■

An Assignment: example

- Construct an assignment as follows:



Max-2-SAT

Max-2-SAT Instance:

- a 2-CNF formula φ

Maximization Problem:

- Find the maximum # of clauses satisfied by an assignment to φ

Max-2-SAT Instance:

- a 2-CNF formula φ and a *threshold* K

Decision Problem:

- Is there an assign. satisfying ≥K clauses of φ?

PAP 186-187

Max2SAT NPC

Theorem:

- Max2SAT is NP-hard

note: clearly Max2SAT∈NP

Proof: 3SAT$\leq_p$Max2SAT

- Replace each $C=(\alpha\vee\beta\vee\gamma)$ of φ w/10 clauses in φ':
  $(\alpha)\wedge(\beta)\wedge(\gamma)\wedge(w_C)\wedge(\neg\alpha\vee\neg\beta)\wedge(\neg\beta\vee\neg\gamma)\wedge(\neg\gamma\vee\neg\alpha)\wedge(\alpha\vee\neg w_C)\wedge(\beta\vee\neg w_C)\wedge(\gamma\vee\neg w_C)$.
- Set $K=7|\varphi|$.

note: w_C="$\alpha=\beta=\gamma=$TRUE?" maximizes satisfiab.

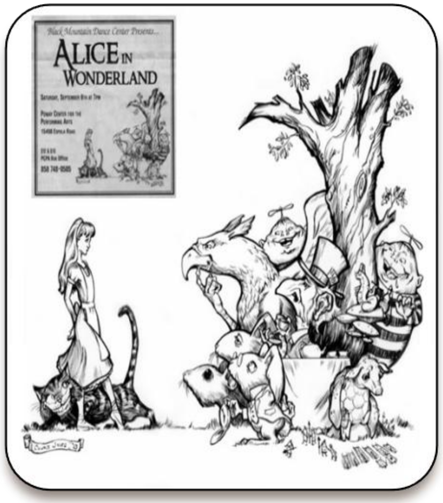
Completeness:

- $C=(\alpha\vee\beta\vee\gamma)$ satisfied $\Rightarrow$ 7/10 clauses satisfied

Soundness:

- $C=(\alpha\vee\beta\vee\gamma)$ unsatisfied $\Rightarrow$ $\leq$ 6/10 clauses satisfied

Synopsis



Discussed variants of SAT

Also: Maximization Problems

Special cases of NPC problems may be in P: SAT vs. 2SAT

Optimization versions of problems in P may be hard: 2SAT vs. Max-2-SAT

WWindex

SAT

Max-2-SAT

NPC

2SAT

Max-2-SAT

NPC

NL Complete

NP-Hard

Complexity Classes

NP

NL

P

L

co-NP

EXPTIME

PSPACE