

COMPLEXITY: Exercise No. 2

1. For each of the following statements, prove, disprove or show that it is an open problem:

- (a) If $L_1, L_2 \in \mathbf{coNP}$ then $L_1 \cap L_2 \in \mathbf{coNP}$. (Test 99)
- (b) If $L \in \mathbf{NP}$, $L_1 \subsetneq L$ and $L_1 \in \mathbf{coNP}$ then $L - L_1 \in \mathbf{NP}$.
- (c) If $L \in \mathbf{NPC}$ then $\{xx : x \in L\} \in \mathbf{NPC}$. (Test 94)

2. Prove (directly, without using Cook's theorem) the polynomial equivalence of the following two problems (namely, show that $\mathbf{DHC} \prec \mathbf{UHC}$ and $\mathbf{UHC} \prec \mathbf{DHC}$):

DIRECTED HAMILTONIAN CYCLE (DHC):

Instance: A directed graph.

Question: Does there exist a directed cycle passing through every vertex exactly once?

UNDIRECTED HAMILTONIAN CYCLE (UHC):

Instance: An undirected graph.

Question: Does there exist an undirected cycle passing through every vertex exactly once?

3. Consider the following problem:

SUBGRAPH ISOMORPHISM (SGI):

Instance: Two graphs, $G = (V, E)$, $H = (U, F)$.

Question: Is there a 1-1 mapping $\phi : U \rightarrow V$, such that if $(u, u') \in F$ then $(\phi(u), \phi(u')) \in E$?

Prove that $\mathbf{UHC} \prec \mathbf{SGI}$.

4. Consider the following problems:

3-SAT:

Instance: A CNF Boolean formula Φ in which each clause has 3 literals.

Question: Is there an assignment that satisfies Φ ?

BALANCED 3-SAT:

Instance: A CNF Boolean formula Φ in which each clause has 3 literals, and every variable in Φ appears negated and unnegated the same number of times.

Question: Is there an assignment that satisfies Φ ?

Prove that $\mathbf{3-SAT} \prec \mathbf{BALANCED 3-SAT}$.

5. Consider the following problems:

SAT:

Instance: A CNF Boolean formula Φ .

Question: Is there an assignment that satisfies Φ ?

1/2 SAT: **Instance:** A CNF formula Φ .

Question: Is there an assignment that satisfies Φ in which exactly half of the variables are TRUE?

Prove that $\mathbf{SAT} \prec \mathbf{1/2 SAT}$.