

COMPLEXITY: Exercise No. 4

1. (Test 95) Is the following problem **NP**-complete?

1/2-INDEPENDENT SET:

Instance: An undirected graph $G = (V, E)$.

Question: Does G have an independent set of size $|V|/2$?

2. (Test 93) Is the following problem **NP**-complete?

IS-CLIQUE:

Instance: An undirected graph G and a positive integer number k .

Question: Does G contain a clique of size k *or* an independent set of size k ?

3. (Test 92) Is the following problem **NP**-complete?

DOMINATING SET (DS):

Instance: An undirected graph $G = (V, E)$ and an integer k .

Question: Does G have a dominating set of size $\leq k$? (a dominating set is a set $U \subseteq V$, such that for every $v \in V \setminus U$ there is $u \in U$ such that $(u, v) \in E$)

4. Is the following problem **NP**-complete?

CONNECTED DOMINATING SET:

Instance: An undirected graph $G = (V, E)$ and a positive integer k .

Question: Does G contain a dominating set S with at most k vertices such that the subgraph of G induced by S (i.e., the graph $G_S = (S, E \cap S \times S)$) is connected?

5. Is the following problem **NP**-complete?

MINIMUM LEAF SPANNING TREE:

Instance: An undirected graph $G = (V, E)$ and a positive integer k .

Question: Is there a spanning tree for G in which the number of leaves is at most k ?

6. (Test 95) Is the following problem **NP**-complete?

MAXIMUM LEAF SPANNING TREE:

Instance: An undirected graph G and a positive integer k .

Question: Is there a spanning tree for G in which the number of leaves is at least k ?