COMPLEXITY: Exercise No. 5

1. Show that MAX-CUT is **NP**-complete when the input graph is simple.

2. (Test 2000) Give a polynomial time algorithm that given a graph $G = (V, E)$ which contains an independent set of size $\frac{3}{4}|V|$ finds an independent set of size at least $\frac{1}{2}|V|$.

3. (Test 98) Consider the following problem:
   MAX-3-CUT:
   Instance: A simple undirected graph $G = (V, E)$ and a positive integer k.
   Question: Is there a partition of V into 3 disjoint sets (V_1, V_2, V_3) such that the number of edges whose endpoints are in different sets is at least k?
   a. Prove that MAX-3-CUT is **NP**-complete.
   b. Give a polynomial time approximation algorithm to the corresponding optimization problem (i.e. the problem of finding a 3-cut of maximum size). What is the approximation ratio?

4. (Test 98) Consider the TSP problem on a complete undirected graph with a length $l_{i,j} \geq 0$ for each edge (i, j). Suppose that the lengths satisfy $l_{i,j} \leq l_{i,k} + 2l_{k,j}$ for all i, j, k.
   Give an approximation algorithm for this problem.
   What is the approximation ratio?

5. Consider the TSP problem on a complete undirected graph with a length $l_{i,j} \geq 0$ for each edge (i, j). Suppose that the lengths satisfy $l_{i,j} \leq l_{i,k} + l_{k,j}$ for all i, j, k. Prove that this problem cannot be approximated to within a factor less than $1 + \frac{1}{n}$, whereas n is the number of vertices in the graph.