

## COMPLEXITY: Exercise No. 7

1. (Test 98) Is the following problem in **NL**?  
Given an undirected graph  $G$ , vertices  $x, y$  from  $G$ , and a positive integer  $k$ , does the shortest path from  $x$  to  $y$  is of length (exactly)  $k$ ?
2. What is the approximation ratio of the greedy algorithm for SET-COVER, when all the sets except one are of size at most  $k$ ?
3. Show that the following problem is PSPACE-complete:  
**Instance:** A deterministic TM  $M$  and an input  $x$  for  $M$ .  
**Question:** Does  $M$  accepts  $x$  without leaving the first  $|x| + 1$  places of the tape?
4. Find a constant  $c$  for which it is NP-hard to approximate VERTEX-COVER to within any constant factor  $< c$ .
5. Show that for any constant  $c > 1$ , there exists a constant  $k$ , such that it is NP-hard to approximate the following problem to within  $c$ .  
**Instance:**  $m$  CNF formulas  $\Phi_1, \Phi_2, \dots, \Phi_m$ , where each formula consists of  $k$  clauses, over the variables  $x_1, x_2, \dots, x_n$ .  
**Problem:** Find an assignment that satisfies as many formulas as possible.  
(Note that  $k$  is constant, and only  $m$  and  $n$  depend on the input.)