COMPLEXITY: Exercise No. 8

1. Prove that if there is a PH-complete problem, then $PH = \Sigma_k$ for some constant k.

2. Prove that if $PH = PSPACE$ then $PH = \Sigma_k$ for some constant k.

3. Prove that $RP \subseteq NP$.

4. Prove that $NP \subseteq PP$.

5. Consider the following alternative definition of ZPP:
   $L \in ZPP$ iff there exists a probabilistic polynomial time TM M that answers TRUE,FALSE or QUIT, and:

   - If $x \in L$ then M always returns TRUE or QUIT.
   - If $x \notin L$ then M always returns FALSE or QUIT.
   - $\forall x \;\; Pr[M(x) = QUIT] \leq \frac{1}{2}$.

   Prove that this definition is equivalent to the definition we saw in class.

6. Prove that $RP \cap co - RP \subseteq ZPP$.