COMPLEXITY: Exercise No. 1

1 Question 1

Prove or disprove:

1. $f(n) = O(n) \Rightarrow 2^{f(n)} = O(2^n)$.
   Answer: False. $f(n) = 2n$ is a counter example (prove!).

2. There is a function f such that $f(n) = O(n^{1+\epsilon})$ for every $\epsilon > 0$ but $f(n) = \omega(n)$.
   Answer: True. $f(n) = n \log n$ satisfies the criterion (prove!).

2 Question 3

Give a NDTM that accepts the *complement* of L_{pal} with space $O(\log n)$ and time $O(n)$.
Answer: We'll use an NDTM with two tapes - the read-only input tape and a working tape. The TM will guess an index $1 \leq i \leq n$, and write it on the working tape, and then check the characters at the i^{th} position from the beginning and from the end. It will accept iff these two characters are different.

3 Question 4

Show that any 1-tape DTM which at any step may move to the left, to the right or stay, can be simulated by a regular 1-tape DTM (where the head has to move at any step).
Answer: We are going to replace each transition in which the head doesn't move, to one step right and one back - left. For any $q \in Q_{old}$ we add a new state q' to Q_{new} (as well as q). We change each transition of the form $\delta(q, a) =< p, b, \downarrow>$ to $\delta(q, a) =< p', b, \rightarrow>$. For each $q \in Q_{old}$ and $a \in \Gamma$ we add the transition $\delta(q', a) =< q, a, \leftarrow>$. Note that the if the old machine works in time $t(n)$ then the new machine takes at most $2 \cdot t(n)$ steps.

4 Question 5

Consider a 1-tape DTM that after each step can move its head either one step right or jump to the first position in the tape (but it can not move one step left). Show that such a machine can simulate an ordinary 1-tape DTM whose time complexity is $t(n)$, in $O(t(n)^2)$ time.
Answer: Let M be ordinary 1-tape DTM. We build a machine M' that simulates M:

We use $\Gamma' = \Gamma \cup \{\#\}$, where the symbol # will be used as a special mark. Whenever the head of M moves right, M' will simulate M naturally. Whenever the head of M moves left, M' moves the entire tape one position to the right, and returns to its original position. More elaborately:

1. M' writes the special mark # on the tape.

2. M' moves right till the right end of the tape, copying every letter it encounters to the position one step right to where it was (including the letter which was previously at the position of the mark). At the end of this stage, all the letters from the mark to the right are shifted one position to the right.

3. M' jumps to the start.

4. M' moves to the right till reaching the mark, again, copying every letter it encounters to the position one step right to where it was (the mark is overwritten by the letter which was previously left to the mark).

Eventually, all the tape is shifted one position to the right, and the head is on the position of the mark, but now the letter appearing at the head position, is the one that was one place left to the mark before that.