Graph Theory

List of theorems whose proofs one should know during the exam

1) A graph is bipartite $\iff$ it contains no odd cycles.
2) The following statements about an n-vertex graph G are equivalent:
   (i) G is a tree, that is, G is acyclic and connected,
   (ii) G is connected and has $n-1$ edges,
   (iii) G is acyclic and has $n-1$ edges,
   (iv) G contains a unique path between every pair of its vertices.
3) Every graph G contains each tree with $\delta(G)$ edges as a subgraph.
4) Mader's theorem: A graph with avg degree $\geqslant 4k$ contains a $(k+1)$-connected subgraph.
5) Whitney's theorem: $\kappa(G) \leqslant \kappa'(G) \leqslant \delta(G)$ for every graph G.
6) Euler's theorem: A connected graph contains an Eulerian circuit $\iff$ all its vertices have even degrees.
7) Dirac's theorem: Every n-vertex graph G with $\delta(G) \geqslant n/2$ is Hamiltonian.
8) The Chvatál–Erdős Theorem: Every graph G with at least three vertices that satisfies $\kappa(G) \geqslant \alpha(G)$ is Hamiltonian.
9) Hall's marriage theorem.
10) Petersen's theorem: Each regular graph with positive even degree has a 2-factor.
11) König's theorem: In every bipartite graph, the largest size of a matching equals the smallest size of a vertex cover.
12) Petersen's theorem: Every 3-regular graph without cut-edges has a perfect matching.
13) Dirac's theorem: Every $(k+1)$-colour-critical graph is k-edge-connected.
14) König's line colouring theorem: If G is bipartite, then $\chi'(G) = \Delta(G)$.
15) The Erdős–Szekeres Theorem: $R(s,t) \leqslant \binom{s+t-2}{s-1}$ for all positive integers s and t.
16) Mantel's theorem: $\mathrm{ex}(n, K_3) = \lfloor n/2 \rfloor \lceil n/2 \rceil$.
17) Euler's formula for connected plane graphs.
18) Heawood's theorem: Every planar graph is five-colourable.