Problem 1. Prove that every two paths of maximum length in a connected graph must have a vertex in common.

Problem 2. Prove that every connected graph has a vertex that is not a cutvertex.

Problem 3. Prove that a graph is 2-connected if and only if for any three vertices $x$, $y$, and $z$, there is a path from $x$ to $z$ that passes through $y$.

Problem 4. Prove that a regular bipartite graph of degree at least 2 does not have a bridge.

Problem 5. Show that every $k$-connected graph with at least $2k$ vertices contains a cycle of length at least $2k$.

Problem 6. Show that if $\kappa'(G) = k \geq 2$, then the deletion of $k$ edges from $G$ results in a graph with at most 2 components.

Problem 7. Let $G$ be a connected graph with $n$ vertices. Prove that $G$ contains a path of length $\min\{2\delta(G), n-1\}$.

Problem 8. A tournament is a complete graph in which each edge $uv$ is given a direction, either from $u$ to $v$ or from $v$ to $u$. Show that a tournament must contain a Hamiltonian path, that is, a directed path through all the vertices. Must it contain a Hamiltonian cycle?

Please do NOT submit written solutions to the following exercises:

Exercise 1. Show that the block graph of a connected graph is a tree.

Exercise 2. Let $G$ be a graph and let $A \subseteq V(G)$. Let $H$ be the graph obtained from $G$ by adding to it a new vertex $v$ with $N_H(v) = A$. Show that $\kappa(H) \geq \min\{|A|, \kappa(G)\}$.

Exercise 3. Prove that $G$ contains the path of length two as an induced subgraph if and only if $G$ is not a union of vertex-disjoint complete graphs.