Graph Theory

Homework assignment #2

Due date: Sunday, December 15, 2019

Problem 1. Show that every k-connected graph with at least $2k$ vertices contains a cycle of length at least $2k$.

Problem 2. Suppose that T is a tree with $2k$ vertices of odd degree. Show that the edge set of T can be decomposed into k paths.

Problem 3. Suppose that a graph G contains two edge-disjoint spanning trees. Show that G contains a spanning Eulerian subgraph, that is, a spanning subgraph that has an Eulerian tour.

Problem 4. Let G be a connected graph with n vertices. Prove that G contains a path of length $\min\{2\delta(G), n-1\}$.

Problem 5. Let G be a non-bipartite graph with n vertices. Show that G has an odd cycle of length at most $\max\big\{3, 2n/\delta(G)\big\}$.

Problem 6. Let G be a bipartite graph with bipartition $V(G) = X \cup Y$ and fix some $A \subseteq X$ and $B \subseteq Y$. Suppose that G contains a matching that covers every vertex of A and also a matching that covers every vertex of B. Show that G contains a matching that covers every vertex in $A \cup B$.

Please do NOT submit written solutions to the following exercises:

Exercise 1. Let G be a graph and let $A \subseteq V(G)$. Let H be the graph obtained from G by adding to it a new vertex v with $N_H(v) = A$. Show that $\kappa(H) \geqslant \min\{|A|, \kappa(G)\}$.

Exercise 2. Prove that G contains the path of length two as an induced subgraph if and only if G is not a union of vertex-disjoint complete graphs.

Exercise 3. Prove that every connected graph contains a vertex that is not a cutvertex.