

Graph Theory

Homework assignment #5

This assignment will not be graded.

Problem 1. Let G be a connected plane graph. Show that $(G^*)^*$ is isomorphic to G .

Problem 2. Suppose that T is a spanning tree of a plane graph G . Show that

$$\{e^* : e \in E(G) \setminus E(T)\}$$

is the edge set of a spanning tree of G^* .

Problem 3. A plane graph is *face-regular* if all its faces have the same degree.

- (a) Characterise all plane graphs that are both regular and face-regular.
- (b) Show that exactly five of these graphs are simple and 3-connected. (These are the Platonic graphs – the skeletons of the five Platonic solids.)

Problem 4. Suppose that G is a 2-connected simple plane graph. Prove that G is bipartite if and only if the boundary of every face of G is an even cycle.