Random graphs

Homework assignment #3

Problem 1. Prove that for every $\varepsilon > 0$ and every sequence $r(n)$ of positive integers satisfying $r(n) \ll n^{1/7}/\log n$, there exists a sequence $p(n)$ of probabilities such that for all sufficiently large n,

$$\Pr\big(\chi(G_{n,p} = r\big) \geqslant 1 - \varepsilon.$$

Problem 2. Prove that if $p \geqslant c/n$ for some constant $c > 1$, then

$$\Pr(G_{n,p} \text{ is bipartite}) = o(1/\log n).$$

Problem 3. Complete the proof of the "switching lemma" stated in class (we omitted the analysis of the "double edge removal switch", see proofs of Claims 3 and 4 there).

Problem 4. Suppose that $d = o(n)$ and let $G_{n,d}$ be the uniformly chosen random d-regular graph with vertex set $\{1, \ldots, n\}$. Prove that there exists a constant C such that a.a.s.

$$\alpha(G_{n,d}) \leqslant \frac{Cn \log n}{d}.$$

Problem 5. Let $G_{n,2}$ denote the uniformly chosen random 2-regular graph with vertex set $\{1, \ldots, n\}$. Prove that there exists a constant C such that

$$\Pr(G_{n,2} \text{ has more than } C \log n \text{ cycles}) = o(1).$$

Problem 6. Let $G_{n,3}$ denote the uniformly chosen random 3-regular graph with vertext set $\{1, \ldots, n\}$. Prove that a.a.s. $G_{n,3}$ is not planar.