Exact orbifold fillings of contact manifolds

–

joint with Zhengyi Zhou

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Plan

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Motivation

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e.g. Mak–Smith '21, Polterovich–Shelukhin '21,
Cristofaro-Gardiner–Humilière–Mak–Seyfaddini–Smith '21

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There are $\mathbb{Z}/2\mathbb{Z}$−graded *groups* $SH^*(W;R)$, $SH^*_+(W;R)$ such that

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$G < U(n)$ such that $\mathbb{C}^n/G$ has isolated singularity. Then, $SH^*(\mathbb{C}^n/G;R) = 0$ if and only if $|G|$ invertible in R.

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W exact orbifold filling of $L(k;1,\ldots,1) = (\mathbb{S}^{2n-1}/(\mathbb{Z}/k\mathbb{Z}), \xi_{st})$ for $n \geq 2$.
Then, $\forall p \in W$ singular: - order of isotropy $|G_p|$ divides $k!$
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Corollary 2

Examples of pairs of contact manifolds with no exact cobordisms in either direction in dim ≥ 5

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Lemma: $f : X \to Y$ has $G_f = \{Id\}$ if $\mathrm{Im}(f) \cap \mathrm{Smooth}(Y) \neq \emptyset$

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Lemma

W exact orbifold filling of smooth contact Y is composition of
- B^{2n}/G_i with isolated singularity, $G_i < U(n)$, for $i \in I$ and $|I| < \infty$,
- exact smooth cobordism $\sqcup_{i\in I}(\mathbb{S}^{2n-1}/G_i, \xi_{st}) \to Y$

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i.e. if $pt \neq 0$ then arrow$=$ Id, if $pt = 0$ then arrow$\in G$ (up to conj!)

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Parametrized by *inertia orbifold*:

$$\Lambda\left(\mathbb{C}^n/G\right) \overset{def}{=} \{(p,(g)) | p \in \mathbb{C}^n/G, (g) \in \mathrm{Conj}(G_p)\}$$

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Definition (Chen–Ruan cohomology)

$$H^*_{CR}(W;R) \overset{def}{=} H^*(W;R) \oplus \bigoplus_{i\in I,(g)\in \mathrm{Conj}^*(G_i)} R[-2\mathrm{age}(g)]$$

where $\mathrm{age}\colon \cup_{i\in I} \mathrm{Conj}^*(G_i) \to \mathbb{Q}$, $(\mathrm{diag}(e^{i2\pi a_1}, \dots, e^{i2\pi a_n})) \mapsto \sum_i a_i$ if $0 \le a_i < 1$.

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Big picture in smooth setting

Different points of view on SH^* of symplectic manifolds with contact boundary:

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- compactification of mod spaces compatible with gluing map (and s.t. $\delta^2 = 0$)

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H_t is *admissible* if:

- H_t is C^2-small Morse and $\mathbb{S}^1-$independent in W
- $H_t = \epsilon \sum_i x_i^2 + C$ near singularities
- $\partial_r H_t > 0$ along Y
- $H_t = h(r)$ on $\widetilde{W} \setminus W$, with $h'(r) = a$ for $r > 1 + \epsilon$, where a not a Reeb period
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$1-$periodic orbits of three types:
- $(p, (g))/C(g)$ as in inertia orbifold
- constant at non-singular Morse-critical points of $H_t|_W$
- non-constant ones (away from singular points)

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Properties:

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- $E(u) = \mathcal{A}(\lim_{s \to -\infty} u) - \mathcal{A}(\lim_{s \to +\infty} u)$
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For x, y $1-$periodic orbits of X_{H_t},

$$M_{x,y} := \widetilde{M}_{x,y}/\mathbb{R}, \quad \widetilde{M}_{x,y} := \{\, u \text{ as in } (*) \mid \lim_{s\to+\infty} u = x,\ \lim_{s\to-\infty} u = y \,\}$$

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Orientations: isotropy $< U(n) \implies$ orientation on constant orbits well def

Compactification of moduli spaces of Floer cylinders

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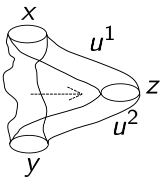

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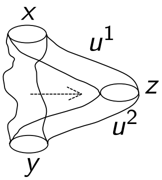

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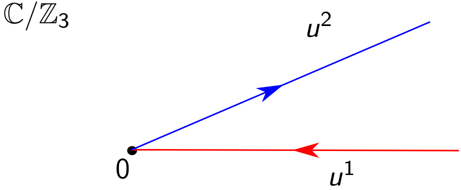

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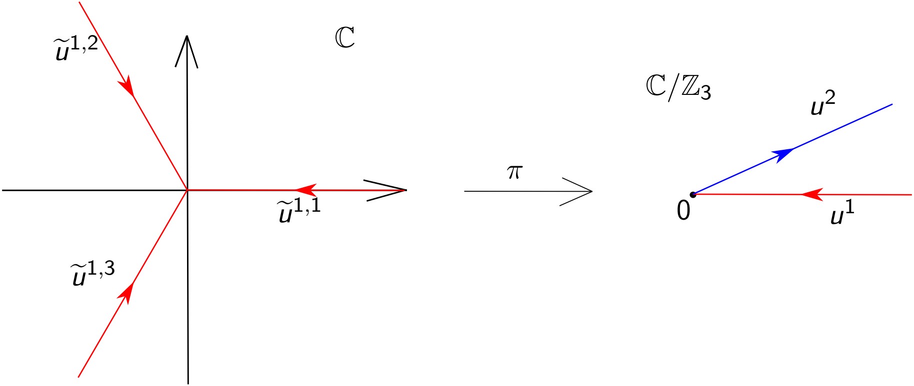

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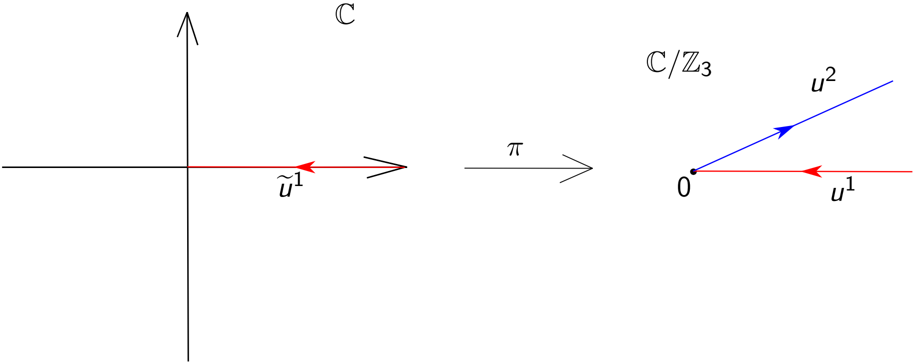

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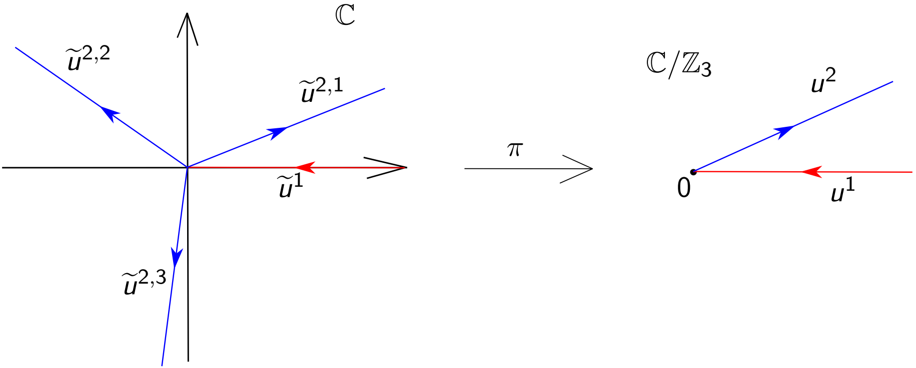

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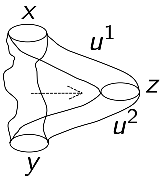

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Right one is by *orbifold fiber product* $\mathcal{M}_{x,y} = M_{x,y} \cup_z M_{x,z} \times_z M_{z,y}$,
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- Viterbo transfer map: if $V \subset W$ exact, $\Phi\colon SH^*(W) \to SH^*(V)$

Plan

$SH^*(\mathbb{C}^n/G; R) = 0$ iff $|G|$ invertible in R

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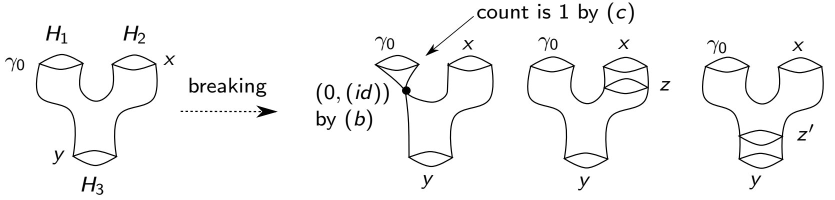

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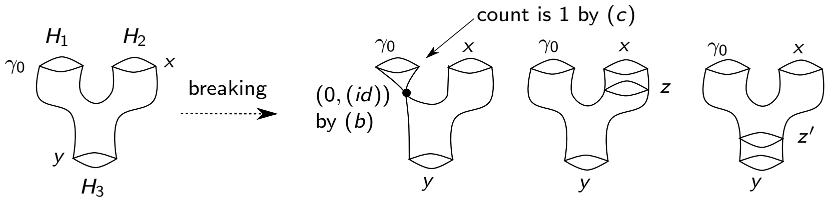

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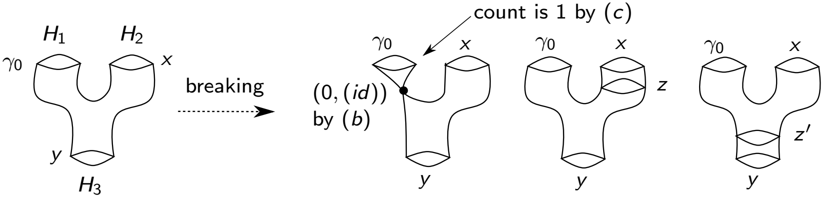

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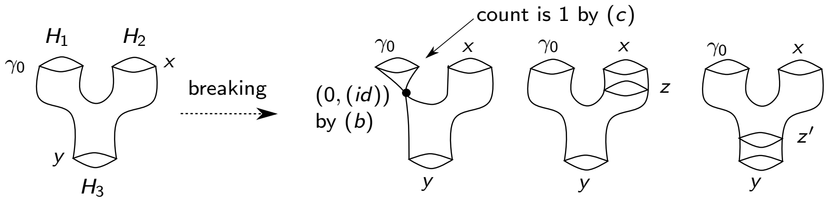

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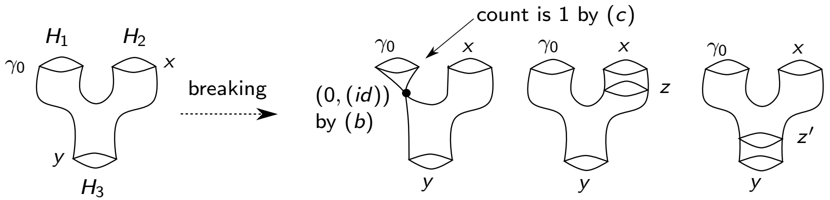

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The end

Thanks for the attention!