Recall: M symplectic manifold

$L \subset M$ Lagrangian with $\pi_2(M, L) = 0$.

(+ tameness at infinity if open)

$K \subset M$ compact subset

$\rightsquigarrow$ $HF^*_M(K; L)$ - not necessarily commutative graded associative algebra over $\Lambda_{\geq 0}$

Construction: Choose $H_1 \leq H_2 \leq \dots$ approximating $\chi(x) = \begin{cases} 0, & x \in K \\ \infty, & x \notin K \end{cases}$ among admissible Hamiltonians

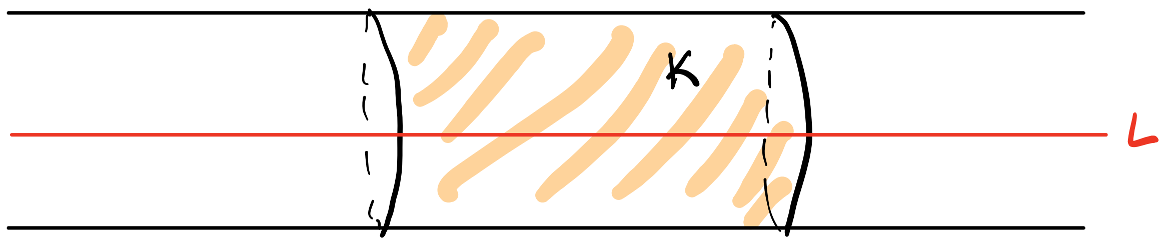
gen'd by
- 1-chords of L
- solutions weighted by top E.

$$\mathcal{C} := CF(H_1; L) \longrightarrow CF(H_2; L) \longrightarrow \dots$$

$$HF^*_M(K; L) := H^*\left(\widehat{\varinjlim \mathcal{C}}\right)^{reg}$$

Example : $\left(\mathbb{R}/\mathbb{Z} \times \mathbb{R}, \; dp\,dq \right)$

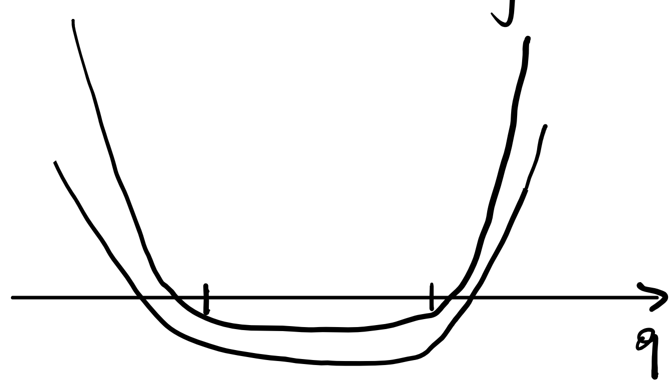
$L = \{0\} \times \mathbb{R}$ $\qquad$ $K = \mathbb{R}/\mathbb{Z} \times [a,b]$ $\swarrow$ $a = b$ allowed



Use particular grading datum

Claim : $HF^*_M(K;L) \cong \widehat{\Lambda_{\geq 0}[x,y]} \big/ xy = T^{b-a}$

Idea :



$$H_i(p,q) = h_i(q)$$

- strictly convex until it becomes linear near ∞
- $X_{H_i} = h_i'(q) \dfrac{\partial}{\partial p} \Rightarrow$ 1-chord $\Leftrightarrow h_i' \in \mathbb{Z}$
- $CF^*(H_i;L) = \bigoplus_{n=1}^{m} x_i^n \oplus \mathbb{1} \oplus \bigoplus_{n=1}^{m} y_i^n$

- All generators supported in degree 0,

in particular $d = 0$.

- Extra Grading by $H_1(M, L) = \mathbb{Z}$ "winding number"

- $x_i^m = x_i * \cdots * x_i$ (same for y_i)

$x_i y_i = y_i x_i = T^{(b-a)+\varepsilon_i}$ $\quad \varepsilon_i \to 0$

- $CF(H_i; L) \to CF(H_{i+1}; L)$

$$x_i \mapsto T^{\delta_i} x_{i+1}$$
$$y_i \mapsto T^{\delta_i} y_{i+1}$$

s.t.

$\sum_{i=0}^{\infty} \delta_i$ converges to $\delta \in \mathbb{R}_{\geq 0}$.

- $\varinjlim \mathcal{C} = \Lambda_{\geq 0}[x, y] \,/\, xy = T^{b-a}$

$\rightsquigarrow$ Desired result.

- Could use S-shaped Hamiltonians.

<u>Generalization</u>: $M = \mathbb{R}^n/\mathbb{Z}^n \times \mathbb{R}^n \quad \sum_{j=1}^{n} dp_j dq_j$

$\pi : M \to \mathbb{R}^n$ projection (convex rational polytope)
$\quad L$

$P \subset \mathbb{R}^n$ compact, conn., intersection of finitely many $\{ l \geq 0 \}$, where could be deg

$l(q) = \sum a_i q_i + b$, $a_i \in \mathbb{Z}$, $b \in \mathbb{R}$.

"integral affine fnc." ← coordinate independent.

$\mathrm{Aff}_{\geq 0}(P) := \{ l : \mathbb{R}^n \to \mathbb{R}$ integral affine such that $l|_P \geq 0 \}$.

$\mathcal{O}_{\mathbb{R}^n}(P) := \Bbbk[\mathrm{Aff}_{\geq 0}(P)]$ (group algebra)

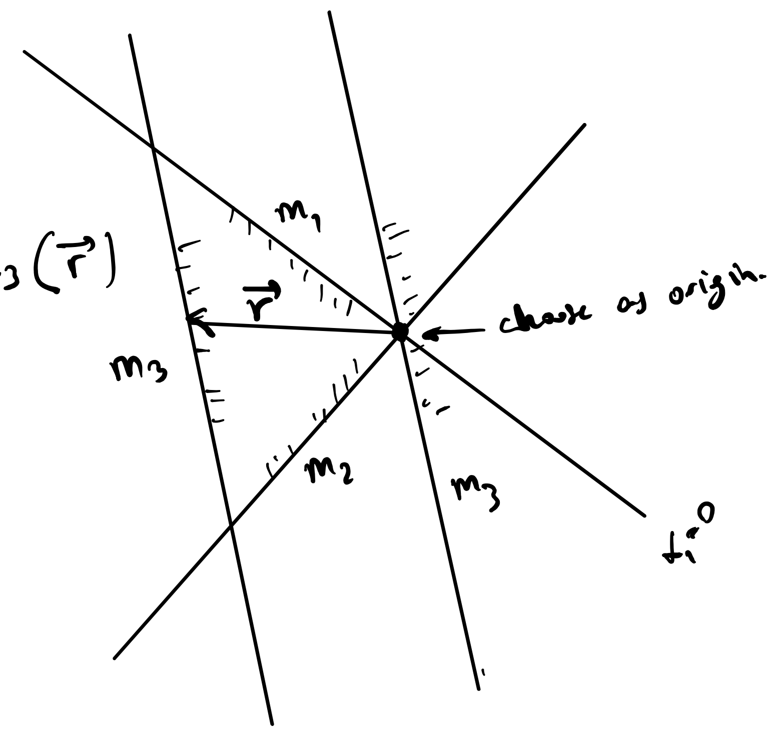
$\mathrm{val}_p : \mathrm{Aff}_{\geq 0}(P) \longrightarrow \mathbb{R} \qquad l \mapsto \min_{p \in P} l(p)$

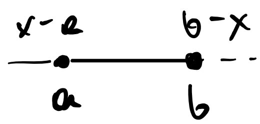
$\hat{\mathcal{O}}_{\mathbb{R}^n}(P) = \widehat{\mathcal{O}_{\mathbb{R}^n}(P)}^{\mathrm{val}_p}$. ($\Bbbk$ commutative ring)

$\ni \sum_{i=1}^{\infty} a_i e^{g_i}$ if $\mathrm{val}(g_i) \to \infty$

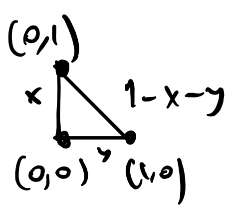
Finding the sum of three affine functions in terms of their vanishing locus if the result is a constant

$\alpha_1 + \alpha_2 + \alpha_3 =$ linear part of $\alpha_3(\vec{r})$



$P =$ 

$\mathcal{O}(P) = \Bbbk[\mathbb{R}_{\geq 0}][x,y] / (xy = T^{b-a})$

$P =$ 

$$\mathcal{O}(P) = \frac{\Bbbk[\mathbb{R}_{\geq 0}][X,Y,Z]}{(XYZ - T)}$$

$\widehat{\mathcal{O}}_{\mathbb{R}^n}(P)$ is canonically a $\Lambda_{\geq 0} = \widehat{\mathbb{k}[\mathbb{R}_{\geq 0}]}$ algebra.

<u>Theorem</u> (essentially Seidel)

$$HF^*_M(\pi^{-1}(P); L) \simeq \begin{cases} \widehat{\mathcal{O}}_{\mathbb{R}^n}(P), & * = 0 \\ 0, & \text{otherwise.} \end{cases}$$

In fact we can again show that, for some acceleration data:

$$\varinjlim \mathcal{C} \cong \mathcal{O}_{\mathbb{R}^n}(P) \underset{\Lambda_{\geq 0}}{\otimes} \Lambda_{>0}$$

The isomorphisms are compatible with restriction maps.

Globalization:

Let Q be an integral affine manifold s.t.

(1) $\pi_2(Q) = 0$

(2) $T^*Q / T^*_{\mathbb{Z}} Q =: X_Q$ is geom. bdd

Rmk: If Q is closed, Markus conjecture would imply (1).

My conjecture: X_Q is geometrically bounded if and only if Q is complete (equivalently geodesically complete)

For $P \subset Q$ convex rational polytope (contained in an integral affine chart by definition), we can define

$$\mathcal{O}_Q(P) \quad \text{and} \quad \hat{\mathcal{O}}_Q(P)$$

using the same formulas.

If $\begin{array}{ccc} \varphi : U & \longrightarrow & V \\ \cap & & \cap \\ Q & & \mathbb{R}^n \end{array}$ is an integral affine chart, then by definition we have canonical isomorphisms

$$\mathcal{O}_Q(P) \cong \mathcal{O}_{\mathbb{R}^n}(\varphi(Q))$$

compatible with restriction maps.

Let $Z_Q \subset X_Q$ be the zero section

Elementary: $\pi_2(Q) = 0 \Rightarrow \pi_2(X_Q, Z_Q) = 0$.

<u>Almost Theorem</u> "locality"

$$HF^*_{X_Q}(\pi^{-1}(p); Z_Q) \cong HF^*_{\underset{X_{\mathbb{R}^n}}{\shortparallel}{\mathbb{R}^n \times (\mathbb{R}/\mathbb{Z})^n}}\big(\pi^{-1}(\varphi(p)); \underset{Z_{\mathbb{R}^n}}{\overset{\shortparallel}{L}}\big)$$

<u>Sketch</u>: (1) Choose S-shaped data

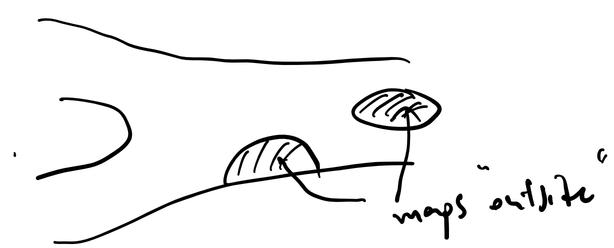
(2) Prove that outside generators die.

(3) Prove that the differential is local

(4) — // — product is local

The key point: $\pi_1(F_b) \hookrightarrow \pi_1(X_Q, Z_Q)$

is injective $\Rightarrow$



can be eliminated.

(seems to require neck stretching for $n \geq 2$)

Corollary (of locality and Seidel's thm)

$$HF^*_{X_Q}(\pi^{-1}(p); z_Q) \cong \hat{\mathcal{O}}_Q(p).$$

compatibly with restriction maps.

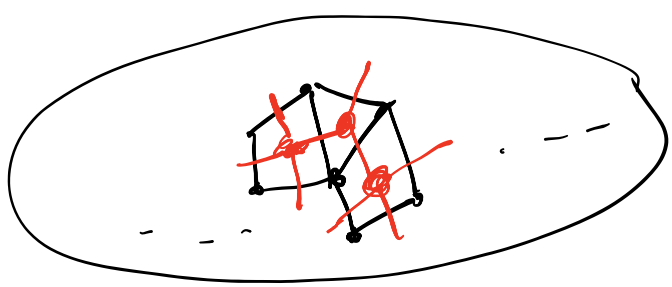
Q also gives rise to a rigid analytic space.

$$\begin{tikzcd}
(\Lambda^*)^n \arrow[r, dashed, "\substack{\text{analytic}\\ \text{automorphism}}"] \arrow[d, "\mathrm{val}"] & (\Lambda^*)^n \arrow[d, "\mathrm{val}"] \\
\mathbb{R}^n \arrow[r, "GL(n,\mathbb{Z}) \ltimes \mathbb{R}^n"'] & \mathbb{R}^n
\end{tikzcd}$$

Glue preimages of coordinate charts using these lifts as transition maps.

$$\begin{tikzcd} X_Q \arrow[d] \arrow[rr, leftrightarrow, "\text{mirror}", "\text{(Abouzaid)}"'] & & Y_Q \arrow[d] \\ Q & & Q \end{tikzcd}$$

<u>Idea</u>: Choosing a decomposition $\mathcal{P}$ of Q into Delzant polytopes upgrades this statement as follows.



Q

There is a symplectic submanifold above that is disjoint from Z_Q and its image under S^1-action near "edges" of $\mathcal{P}$.

(assuming certain integrality $[D_{\mathcal{P}}] = PD[\omega]$)

We can define a Reynaud model $\mathcal{Y}_P$ of $\mathcal{Y}_Q$ whose special fibre is mirror to $X_Q \setminus D_P$.

[Reynaud model means a formal scheme over $\Lambda_{\geq 0}$ whose (Reynaud) generic fibre is the rigid analytic space.]

<u>Rmk</u>: subdivision $\longleftrightarrow$ blow-up in the special fibre

simplicies $\longleftrightarrow$ nc

$\mathcal{Y}_P$ is constructed by gluing

$$\operatorname{Spf} \widehat{\mathcal{O}}(P_i) \quad \text{along}$$

$$\operatorname{Spf} \widehat{\mathcal{O}}(P_i \cap P_j) \subset \operatorname{Spf} \widehat{\mathcal{O}}(P_i)$$

There are two key statements.

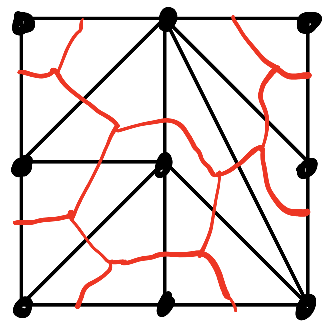
<u>Prop 1</u>: $\mathcal{O}_{\mathbb{R}^n}(P)$ is finitely presented over $\mathbb{k}[\mathbb{R}_{\geq 0}]$.

If P is Delzant, then "the codim 1 faces generate"

<u>Prop 2</u>: Let P Delzant & $F \subset P$ be a face, then $\mathcal{O}_{\mathbb{R}^n}(P) \to \mathcal{O}_{\mathbb{R}^n}(F)$ is the localization map at the generators of the codimension 1 faces that contain F.

Example :

particular grading datum



$\rightsquigarrow (\mathbb{R}/\mathbb{Z})^2 \times (\mathbb{R}/2\mathbb{Z})^2$

fibres ↓ base ↓

zero section

Relative HF for each

triangle $\cong \Lambda_{\geq 0}\{\{X, Y, Z\}\}/XYZ - T$

$\cong \left(\mathbb{k}[[T]]\{\{X, Y, Z\}\}/XYZ - T\right) \otimes \Lambda_{\geq 0}$

can also glue there

The special fibre of the Reynaud model is obtained by gluing $XYZ = 0$ inside $\mathbb{k}^3$ along $\{X \neq 0\}$, $\{Y \neq 0\}$, $\{Z \neq 0\}$ according to the shared edges.

More topics that could be discussed:

Why can't we glue $\mathrm{Spec}(\mathcal{O}_{\mathcal{B}}(P_i))$?

Generalization to ...

1-d example

HMS functors / local generation

Auroux, Ishii - Ueda conjectures. local to global approach.