Dimers, networks, and integrable systems.

Integrable system

M – poisson mfld.

$\{f_i\}$ – fns.

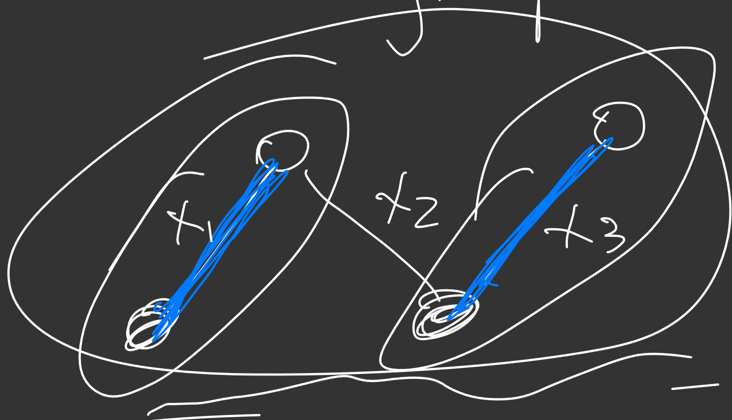
$\{f_i, f_j\} = 0$

Goncharov-Kenyon.

dimer model

weighed Bipartite graph

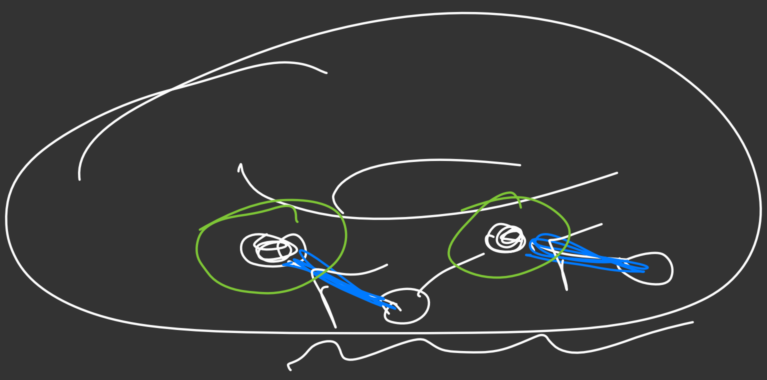
$X_1 X_3$



dimer cover

$=$ perfect matching

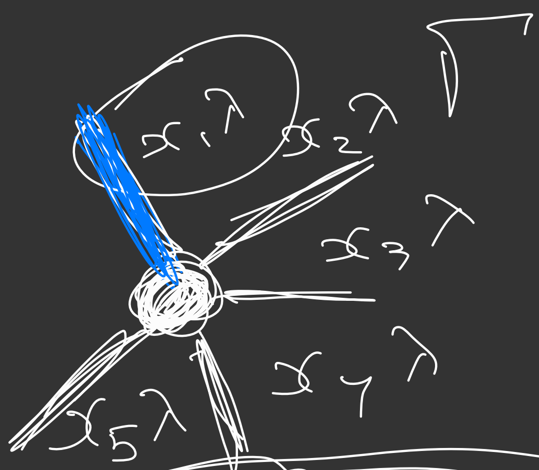
weight of perfect matching $=$ product of weight weighted bipartite graph on torus



matching ⇝ homology class

$[\gamma] \in H_1(T^2)$

$$H_\gamma = \sum_{\text{matchings} \in [\gamma]} \text{weights}$$

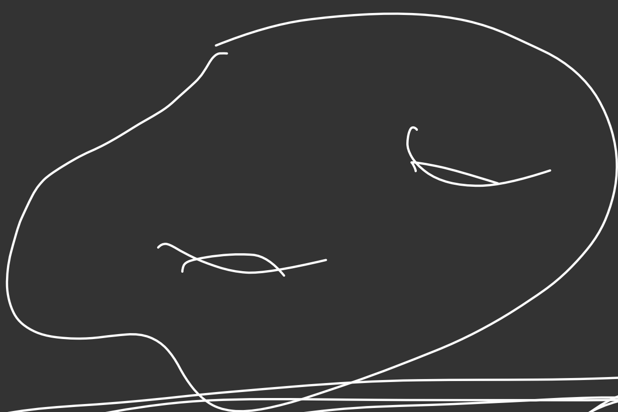


gauge transf. $H^1(\Gamma, \mathbb{C}^*)$

edge weight / gauge transf.

$$H^1(\Gamma, \mathbb{C}^*)$$

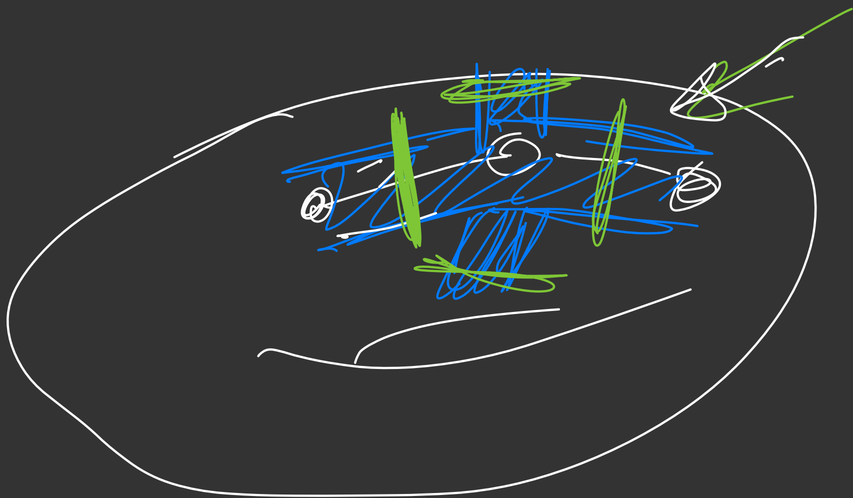
$$H^1(\Gamma, \mathbb{C}^*) \to \left(H_{\gamma_1}, H_{\gamma_2}, \dots \right)$$



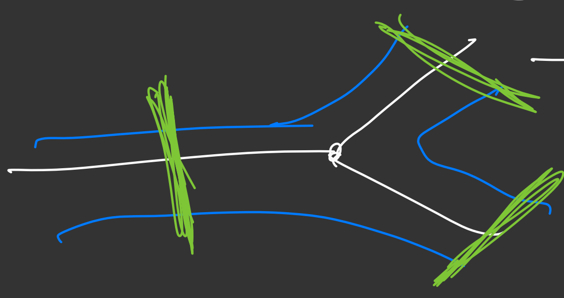
S

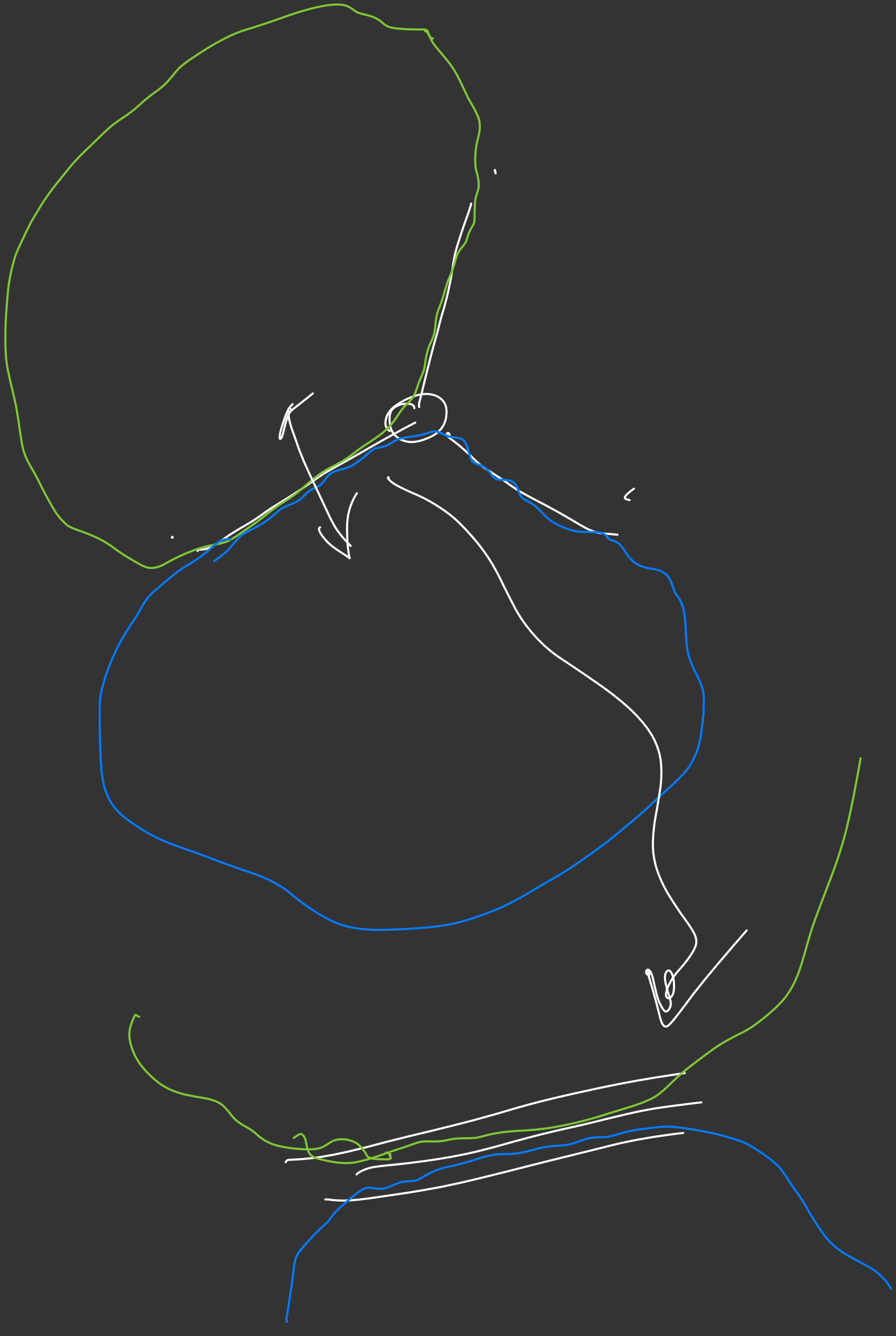
$$\{a, b\} = \langle a, b \rangle \, ab$$

$$H^1(S, \dots)$$



Th. H_γ on $H^1(\Gamma, \mathbb{C}^*)$ Poisson commute.

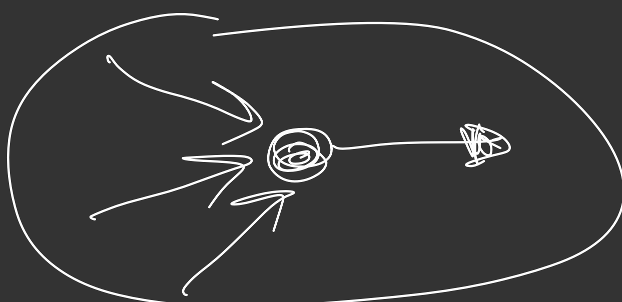


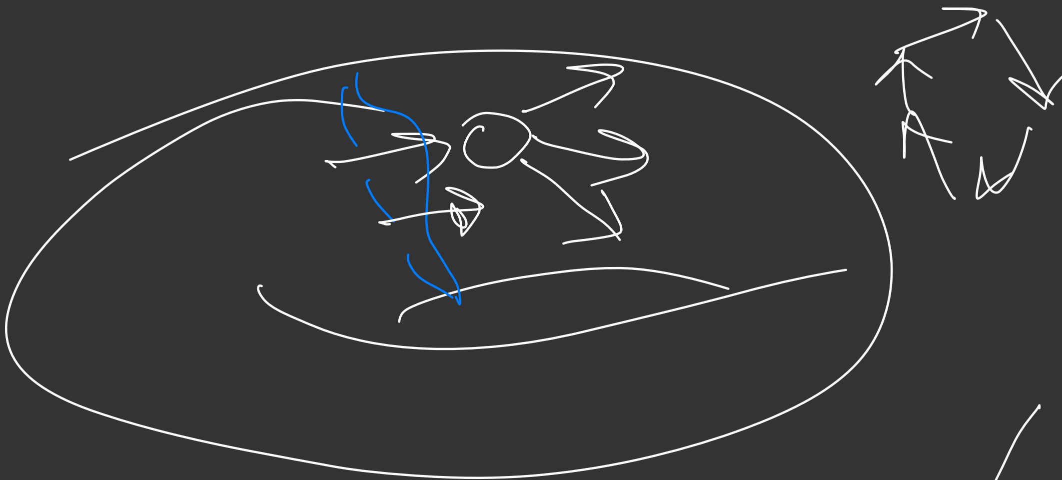


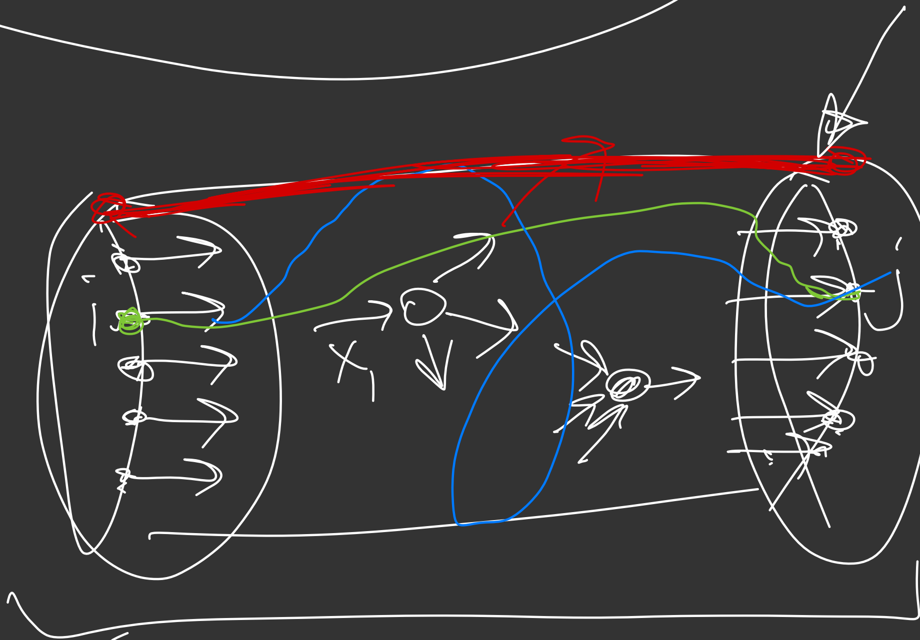
Perfect networks

Postnikov

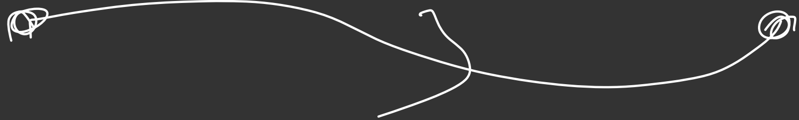
GSV, GSTV







$m_{ij}(\lambda) = \sum \pm$ weight of paths $\cdot\, \lambda^{j}$ going $i \to j$



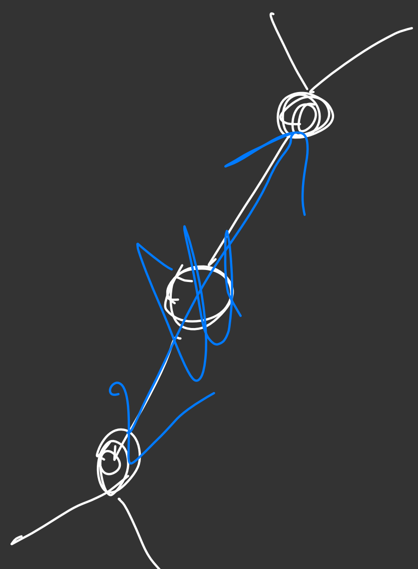
$M = (m_{ij}(\lambda))$

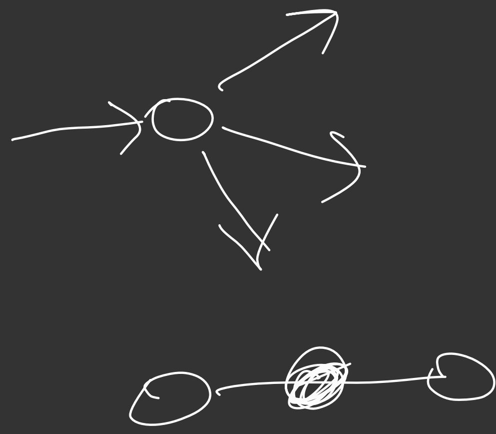
GSTV

$\det(M(\lambda) - \mu\,\mathrm{Id})$

Bipartite graph on torus

perfect network on a torus





$$K(\lambda,\mu) = \sum \pm H(i,j)\,\lambda^{i}\mu^{j}$$

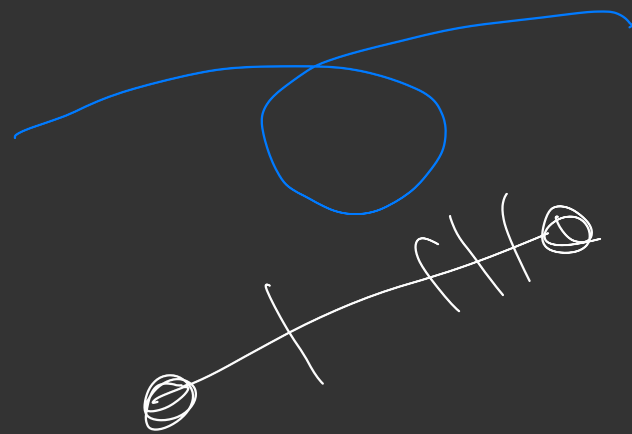
$$(i,j) \in H(T^2)$$

$$+ \quad + \quad + \quad -$$

$$K(\lambda,\mu)$$

$\det K$

$$\det(\mathrm{Id} + \mu M(\lambda))$$

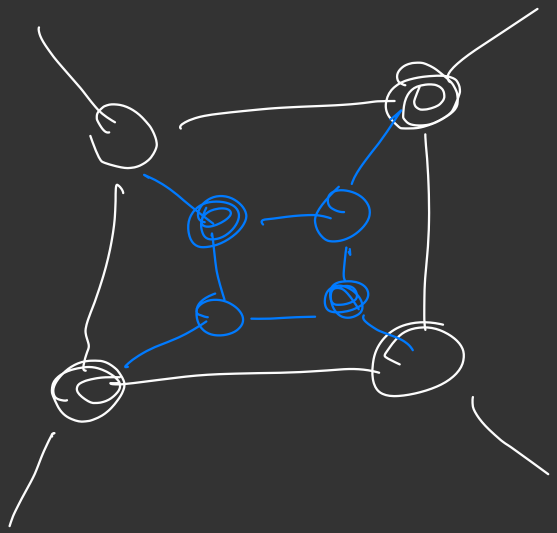


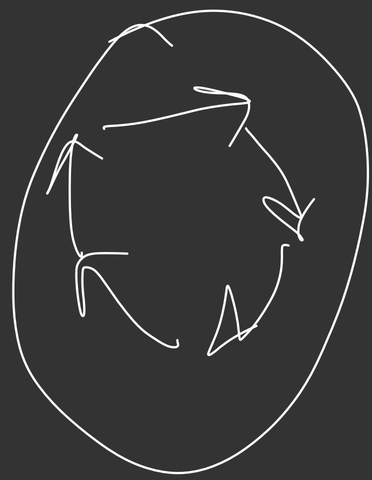
Th.

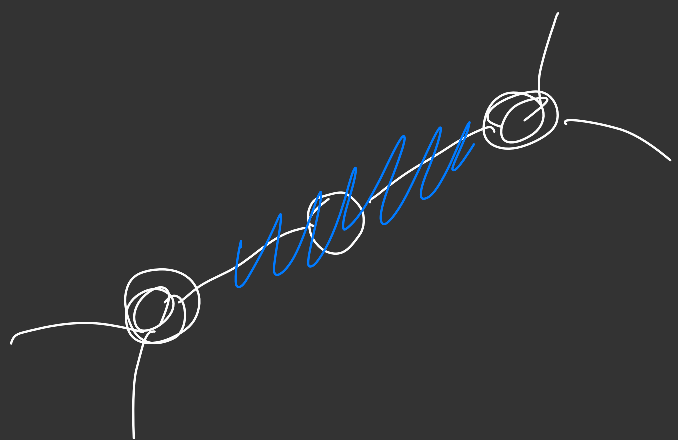
$$\det(\mathrm{Id} + \mu M(\lambda))$$

$$= K(\lambda, \mu)$$

$$K(\lambda, 0)$$







$$H^1(\Gamma, \mathbb{C}^*)$$