$$(X^{CY}, \omega) \quad \longleftrightarrow \quad (Y, J)$$

Enumerative closed GW inVs. $\longleftrightarrow$ Period integrals

homological: $\quad FUK(X) \cong D^b coh(Y)$

$\{ \} GPS 15$

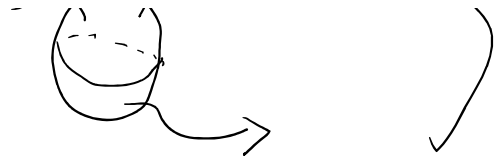Extended: open Gromov-Witten $\longleftrightarrow$ "extended period integrals

$[L] \in H_2(X)$

choose $C \subseteq X, \partial C = L$.

$\# \left\{ \begin{array}{c} c_1 \subseteq X \\ c_2 \subseteq X \\ c_3 \subseteq X \end{array} \right\}$ L

$+ \# \left\{ \begin{array}{c} C \\ c_1 \\ c_2 \\ c_3 \end{array} \right\} = OGW(c_1, c_2, c_3)$

$$QH^*(X) := H_*(X) \otimes \Lambda_q$$
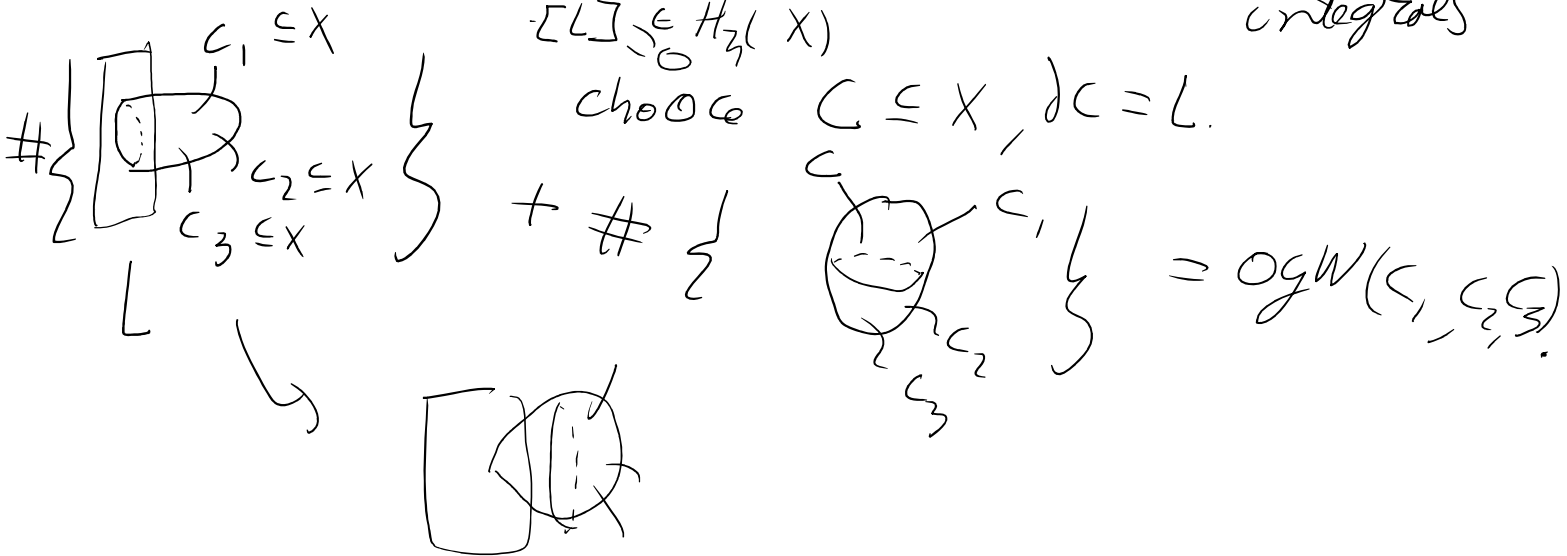
novikov-field

$$\nabla_{q \frac{d}{dq}}(C) = q \frac{d}{dq}(C) + \boxed{[\omega]} \star C.$$

$PD(\omega)$ C

quantum VHS

+ (filtration)

$\underline{gPS}$: $QH^*(X)$ $\leadsto$ closed Gromov-Witten

$\overset{OC}{\nwarrow}$ isomorphism of VHS

$Fuk(X) \leadsto HC^-_*(Fuk(X))$ VHS

S.T : $\overset{given\ L \leq X}{QH^*(X,L)} = H_*\left(Cone(\mathbb{Z} \to C_*(X))\right) \otimes \Lambda$

$\underset{1 \to L}{}$

$\nabla_{q\frac{d}{dq}}(a, \alpha) = \left(q\frac{d}{dq}(a), \nabla_{q\frac{d}{dq}}(\alpha) + a \circ \boxed{} \right)$

$PD(\omega)$

L

VHS.

$\underline{thm(H.)}$: $QH^*(X,L)$ $\leadsto$ $\overset{1-Point}{OGW}$ invariants.

$F: * \longrightarrow Fuk(X)$

$\curvearrowright$

$\wedge$

$* \longmapsto L$

$1 \longmapsto e_L$

$HC^-_*(Fuk(x), L) = H_*\left(Cone\left(F_* : CC^-_*(*) \to CC^-_*(Fuk(x))\right)\right)$

$\underline{lemma}$: can   equip this with a VHS.

conj: $\exists \, OC_L : \underline{HC_*(Fuk(X), L)} \to QH^*(X, L)$

isomorphism of VHS.

thm: (techincal assumptions: same as S.T. to define ogw)

$\exists \, OC_L : \underline{HC_*(CF^*(L,L), L)} \to QH^*(X, L)$

morphism of VHS.

assuming conjecture:

$Fuk(X) \rightsquigarrow$ 1-point ogw of X.