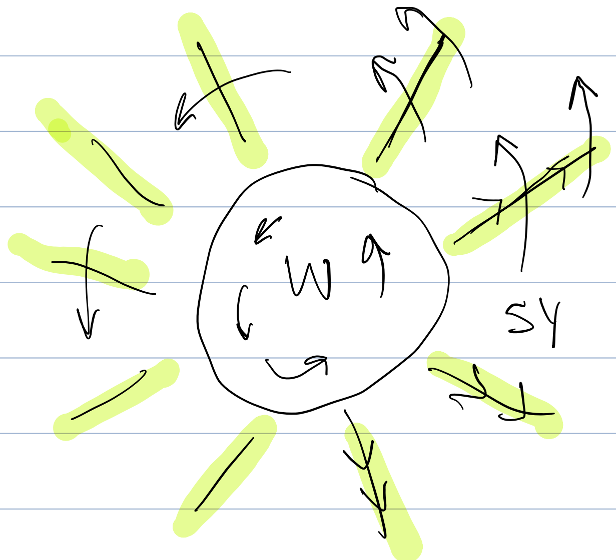
ZOOMINAR! arXiv: 2311.18267

EXTENSIBLE POSITIVE LOOPS and vanishing of $\underset{\sim}{SH}$ (symplectic cohomology).

Joint w/ Jakob Hedicke, Eric Kilgore.

$\boxed{\text{Merry – Uljarević. 2019}}$



arbitrary contact isotopy in end

$\rho_r : SY \to SY$

$\boxed{\varphi_t} \circ \rho_r = \rho_r \circ \varphi_t$

φ_t induces a contact isotopy on Y

application: if $\exists\ \varphi_t$ cont. iso. $\varphi_0 = id = \varphi_1$

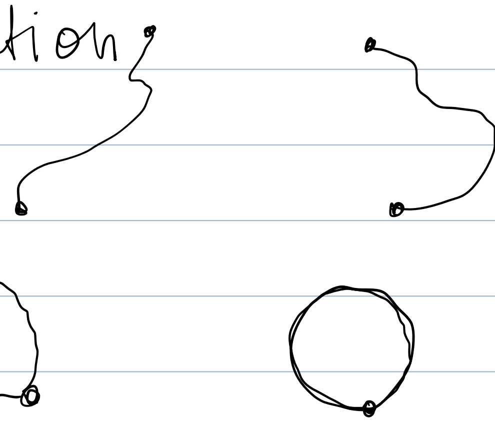
which is <u>positive</u> [EP2000]

which is <u>extensible</u> $\varphi_0 = \varphi_1$ of Hamiltonians in W

Then $\boxed{\dim SH(W) \leq \underline{\dim H^*(W)}}$

$$\boxed{\mathbb{R} : \mathrm{Ham}(W) \longrightarrow \mathrm{Cont}(Y)}$$

serre fibration:



$SH(\mathbb{C}^n) = 0$ $\quad$ $\mathbb{C}^n$ has a pos. ext. loop.

φ_t^k more and more positive

$$HF(\varphi_t^k) \to HF(\varphi_t^{k+1}) \to \cdots$$

1 dim $\qquad$ 1 dim

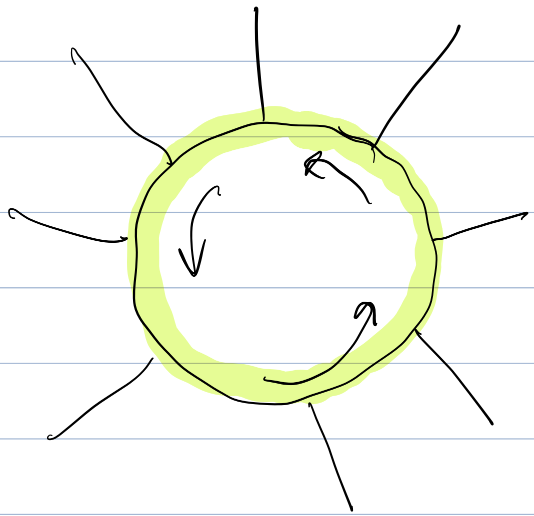
$$\boxed{\cdot} \xrightarrow{0} \boxed{\cdot} \xrightarrow{0} \boxed{\cdot}$$

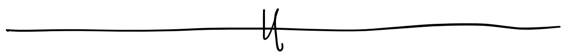
CCDR 19 $\quad$ if Y has a positive contractible loop then $SH(W) = 0$

Albers – Merry $\quad$ if Y has a pos. contr. loop + SFC + then $RFH(W) = 0$

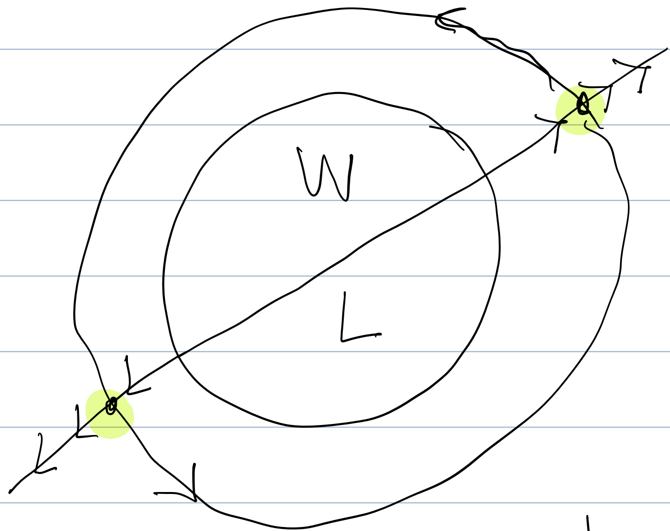
Ritter 2014 – 2016

assumes contact isotopies are strict for some contact form α.





we also develop a relative version



Z tangent to TL

$L \cap Y = \underbrace{\Lambda}$ Legendrian

$$\mathrm{Lag}(W) \xrightarrow{\mathbb{R}} \mathrm{Leg}(Y)$$

$\boxed{\text{Theorem}}$ if Λ admits a pos. extensible loop then $HW(L) = 0$.

note $SH(W) = 0 \implies HW(L) = 0.$

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corollary recover famous result

$$SH(W \times \mathbb{C}) = 0.$$

[EP2000]

corollary suppose Λ is fillable and admits positive extensible loop. Then Λ has Reeb chord $\forall \alpha$.

$HW(L) = 0 \implies \Lambda$ has Reeb chord.

(AFM) suppose Y admits a pos. loop in $\mathrm{Cont}(Y)$. Then Y admits Reeb ORBIT for every α.

Extensibility is also related to

S.M.C. and exotic lagrangian fillings.

$$\mathrm{Ham}_c(W) \longrightarrow \mathrm{Ham}(W) \xrightarrow{\mathbb{R}} \mathrm{Cont}(Y)$$

$$\underbrace{\mathrm{Ham}_c(W)}_{} = \mathrm{Ham}(W) \cap \mathrm{Diff}_c(W).$$

$$\delta : \pi_1 \mathrm{Cont}(Y) \longrightarrow \pi_0 \mathrm{Ham}_c(W).$$

a loop φ_t is extensible iff $\delta[\varphi_t] = [id]$.

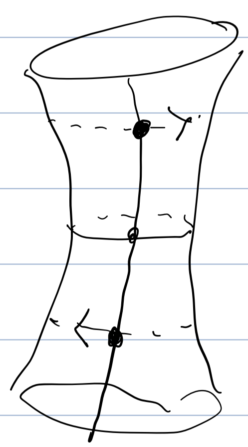
non-extensibility $\Rightarrow$ $\exists$ nontrivial elements in $\pi_0 \mathrm{Ham}_c(W)$.

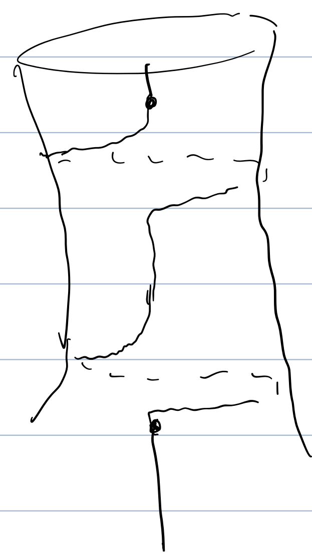
Theorem if $SH(W) \neq 0$ and Y admits a positive loop, φ_t

then $\delta[\varphi_{kt}]$ are distinct in $\pi_0 \mathrm{Symp}_c(W)$

[W to be atoroidal]

T^*S^1





$\square$

Proof

key idea is the so-called twisting trick

[MU19], [Ritter 14], [Ritter 16], ... [Ulj 17], ...

compares naturality transfo. versus continuation

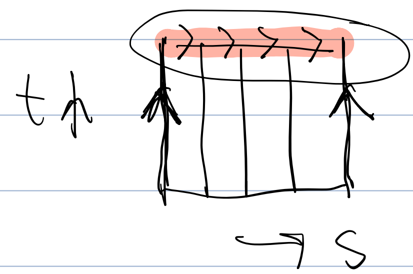
given ψ_t Ham isot. associate $HF(\psi_t)$

$HF(\psi_t) = \mathbb{Z}/2 \langle$ fixed points of $\psi_1 \rangle$.

ψ_1 should have no discrim. points @ ∞.

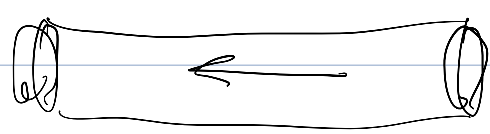
given a nonnegative path of isotopies $\psi_{s,t}$

$HF(\psi_{0,t}) \to HF(\psi_{1,t})$

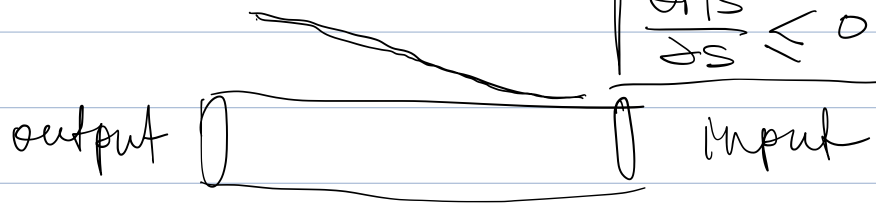
$IR(\psi_{s,t})$  $\in$ $\mathrm{Cont}(Y)$

$w : \mathbb{C} \to W$ $\quad \partial_s w + J(w)\partial_t w = 0$

twisted periodic $\psi_{s,1}(w(s,t+1)) = w(s,t)$.

[DS 93] output  input

$$\boxed{\frac{\partial H_s}{\partial s} \leq 0}$$



cont. ☑   naturality ☑

if φ_t is a loop in $Ham(W)$ based at id.

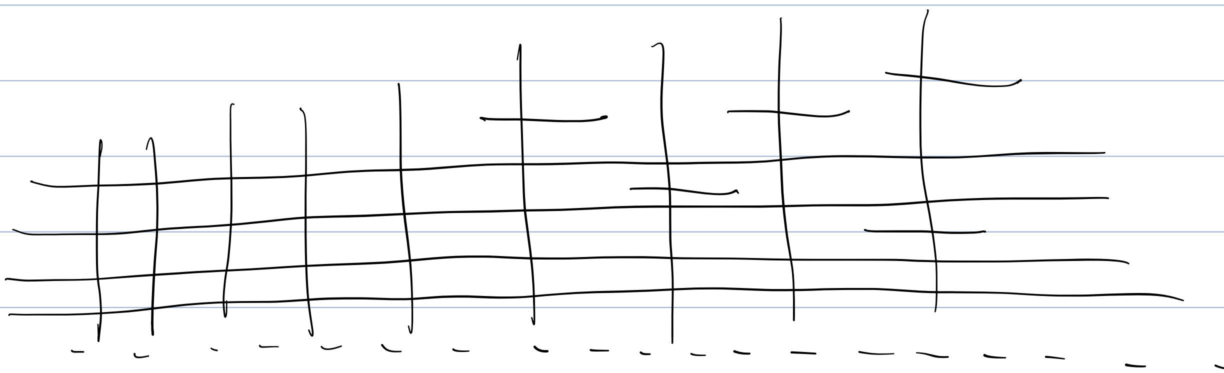
Then $HF(\varphi_t \psi_t) \xleftarrow{h} HF(\psi_t)$

naturality commutes w/ continuation.

Lemma if $\exists$ pos. Ext. loop, take ψ_t Ham isot.

$$\boxed{HF(\psi_t) \twoheadrightarrow SH(W)}$$

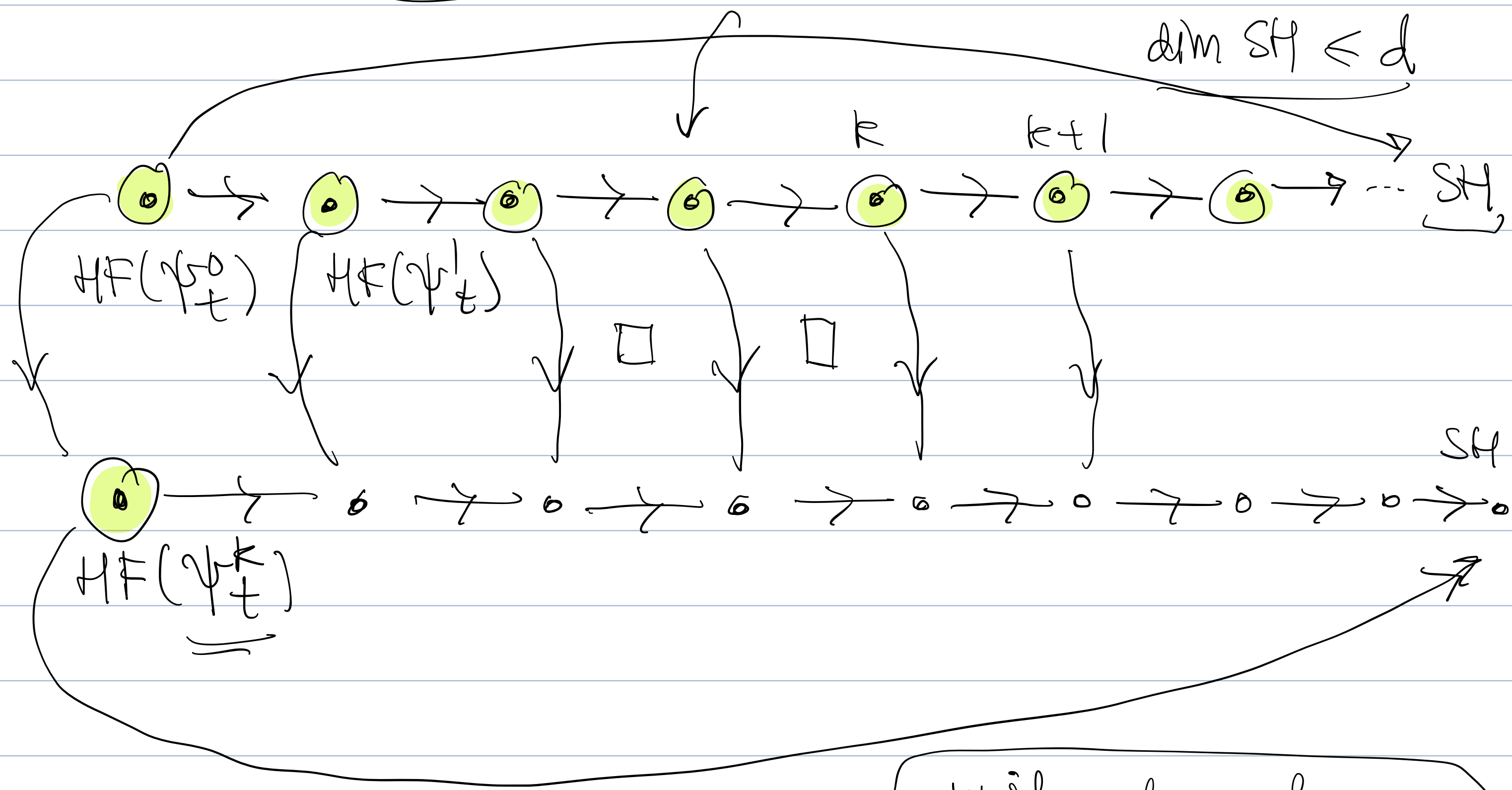
colimit of $HF(\tilde{\psi}_t)$ as $\tilde{\psi}_t$ gets more pos.



proof take $\varphi_t \psi_t^0 = \psi_t^1$

$$\psi_t^{k+1} = \varphi_t \psi_t^k$$

$$\underbrace{\psi^k_{s,t}} = \underbrace{\varphi_{st}\,\psi^k_t}$$



unit element

if $SH(W) \neq 0$ then 1 in SH does NOT lie in the image of $HF(R^{\alpha}_{-\epsilon t})$

$\therefore$ $\nexists$ pos. ext. loop. if $SH \neq 0$.