Assignment 4 - Computational Geometry (0368-4211)

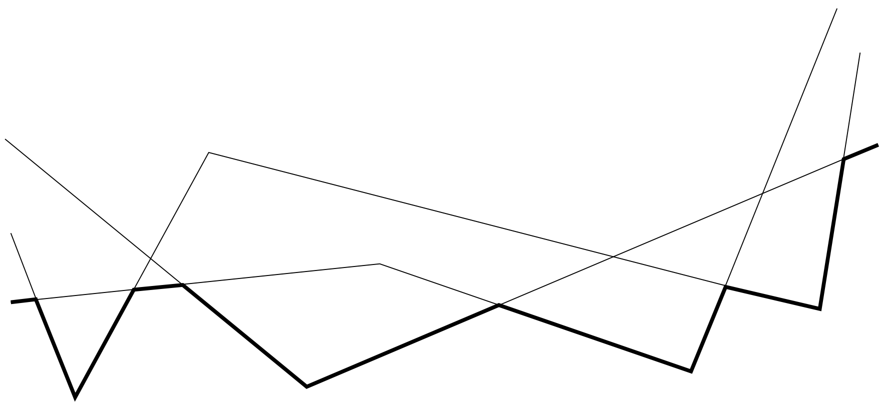
Due: June 15, 2015

Problem 1

Let T be a set of n triangles in the plane.

(a) Describe in the dual plane the structure of the set K of points dual to all the lines that intersect *all* the triangles.

(b) The following is known: The complexity (number of edges and vertices) of the lower (or upper) envelope of n line segments and rays in the plane is $O(n\alpha(n))$, where $\alpha(n)$ is the inverse Ackermann function, which grows extremely slowly with n. Since $\alpha(n) \ll \log n$, use instead the weaker (but simpler) bound of $O(n \log n)$ for the complexity of the envelope.



Lower Envelope

Assuming this, give a near-linear algorithm for computing K. (**Hint:** Use a divide-and-conquer approach to construct the relevant envelopes; if you solved (a) correctly, the envelopes are there...)

(c) Assume now that all the triangles of T contain the origin in their interior. Describe the structure of the set M of points dual to the lines that *miss* all the triangles in T. Show that this set has a near-linear complexity and compute it in near-linear time. I prefer to see two solutions: One that does all the analysis in the primal plane, and the other that maps each triangle $t_i \in T$ to the set M_i of points dual to the lines that intersect t_i, and then reasons directly about the sets M_i.

Problem 2

Given two sets $A = \{p_1, \ldots, p_n\}$, $B = \{q_1, \ldots, q_m\}$ of points in the plane. Determine in linear time whether the two sets can be separated from one another by a line, and, if so, produce such a separating line. How can the problem be extended into higher dimensions and how efficiently can it be solved?

Problem 3

Given n half-planes of the form $a_i x + b_i y + c_i \geq 0$, for $i = 1, \ldots, n$. Find, in $O(n)$ time, the largest circle that is fully contained in their intersection.

Problem 4

Use duality to solve efficiently the following problem: Given a set S of n points in the plane, find the triangle of smallest area whose vertices belong to S. (As a special case, determine whether S has three collinear points—they define a triangle of zero area.) (**Hint:** Prove that if abc is the smallest such triangle, then, up to some permutation of a, b, c, the line c^* lies immediately above or below the point of intersection of the lines a^*, b^*. Use this observation to get an $O(n^2 \log n)$ solution, using, e.g., line-sweeping.)