8 Homework

Exercise 8.1. 1. Show that any compact convex set with non-empty interior has an ellipsoid of maximal volume contained in it.

2. Show that any compact convex set with non-empty interior has a parallelopotope of maximal volume contained in it.

3. Show that any compact convex set with non-empty interior has a ellipsoid of minimal volume containing it.

4. Show that any compact convex set with non-empty interior has a parallelopotope of minimal volume containing it.

Exercise 8.2. Find an inequality between Hausdorff between two convex bodies and their Banach-Mazur distances (which may involve parameters of the bodies such as in-radius). Show that these distances can be significantly different even in a fixed dimension.

Exercise 8.3. Show that every sequence of compact convex sets has a converging subsequence with respect to the Banach Mazur distance (that is, $d_{BM}(K_m, K) \to 1$).

Exercise 8.4. Find the geometric distance between $\ell_p^n$ and $\ell_q^n$; give an example for $n = 2$ where the geometric distance is different from the Banach Mazur distance.

Exercise 8.5. Show the following fact: Let $K$ be a convex body in $\mathbb{R}^n$. An ellipsoid $E$ is the ellipsoid of maximal volume inside $K$ if and only if $E^\circ$ is the ellipsoid of minimal volume outside $K^\circ$. 