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Robust Low Speed Preconditioning for Viscous High Lift Flows

E. Turkel
Tel Aviv University, Israel
and *ICASE, NASA Langley Research Center, Hampton, VA 23681*

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An analysis of preconditioning for viscous compressible flow is presented. We consider the effect of the preconditioning as the Mach number goes to zero but the cell Reynolds number remains finite. Adjustments to the preconditioning parameters are suggested. A range of high angle of attack computations, including turbulence models, in both two and three space dimensions are presented to validate the effectiveness of the improved code.

Introduction

METHODS for preconditioning the low speed Euler equations have been available for about twenty years. In spite of these years of development and analysis there still exists difficulties with the robustness of these techniques. This manifests itself in that some of the parameters of the preconditioning matrix are problem dependent. In particular the parameter β might depend on the local Mach number. Because of the stagnation points it is necessary to provide a lower bound on β to prevent it from becoming too small. Unfortunately, the cutoff is usually quite large and problem dependent. Hence, in practice β is a problem dependent constant. In addition the theory of preconditioning is mainly based on properties of the inviscid Euler equation even though many of the applications are for viscous flow.

In describing the fluid dynamic equations one has the choice of several sets of dependent variables. This is true both for the analysis and for the computer implementation. Each set of variables has its advantages and disadvantages. In the course of this study we shall refer to the following sets of variables.

1. Conservation variables
$$w_c = (\rho, \rho u, \rho v, \rho w, \rho E)$$
2. Primitive entropy variables
$$dw_0 = (\frac{dp}{\rho a}, du, dv, dw, dp - a^2 d\rho)$$
3. Primitive pressure-temperature variables
$$w_T = (p, u, v, w, T)$$
4. Viscous variables in differential form
$$dw_N = (\frac{a}{\sqrt{\gamma}\rho}d\rho, du, dv, dw, \frac{R\sqrt{\frac{\gamma}{\gamma-1}}}{a}dT)$$

*Professor, Department of Mathematics, Associate Fellow

The conservation variables are necessary for the equations to be expressed in divergence form. This is important to get the correct jump conditions across shocks. Hence, the space flux terms will always be evaluated in conservation variables. If other sets of variables are used then the fluxes will be converted to these other sets after they are evaluated in conservation form. This will guarantee that the steady state solution is achieved in conservation variables and so the correct shock conditions are obtained in the steady state.

The entropy variables, w_o, give rise to sparse symmetric matrix coefficients for the Euler equations. Hence, this set is useful for the construction and analysis of preconditioners for the Euler equations. The temperature variables, w_T, are convenient for viscous flow. Alternatively, the enthalpy can replace the temperature.[14] Viscous, incompressible flow is frequently expressed in terms of these variables. Hence, it is convenient to use these variables to examine the convergence of solutions of the viscous compressible equations to the viscous incompressible equations as the Mach number goes to zero. This set is commonly used in many codes.[2,13,17] The viscous variables, w_N, are useful for the analysis of the compressible linearized Navier-Stokes equations as the equations are symmetric in these variables and the viscous matrices are diagonal.

Given the equations in quasi-linear form for one set of variables it is straightforward to convert it to a different set of variables. Consider the preconditioned inviscid equations for U,

$$P^{-1}U_t + AU_x + BU_y + CU_z = 0 \qquad (1)$$

Then in W variables the equations are given by

$$P_1^{-1}W_t + A_1W_x + B_1W_y + C_1W_z = 0 \qquad (2)$$

where

$$P_1 = \frac{\partial W}{\partial U} P \frac{\partial U}{\partial W}$$

and a similar relation for A_1, B_1, C_1.

In particular for w_0 variables we choose[9–11]

$$\mathbf{P_0}^{-1} = \begin{pmatrix} \frac{1}{\beta^2} & 0 & 0 & 0 & \delta \\ \frac{\alpha u}{a\beta^2} & 1 & 0 & 0 & 0 \\ \frac{\alpha v}{a\beta^2} & 0 & 1 & 0 & 0 \\ \frac{\alpha w}{a\beta^2} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

which has the free parameters β,α and δ. We choose

$$\beta^2 = \max\left(\min(K_1 M^2, \beta^2_{\min}), 1\right)$$
$$\beta_{\min} = K_2 M_{\text{ref}} \quad K_1 \sim 1.$$

In generalized two dimensional coordinates M^2 is given by

$$M_\xi = \frac{u\xi_x + v\xi_y}{a\sqrt{\xi_x^2 + \xi_y^2}}, \quad M_\eta = \frac{u\eta_x + v\eta_y}{a\sqrt{\eta_x^2 + \eta_y^2}},$$
$$M^2 = M_\xi^2 + M_\eta^2.$$

If β^2=1 and α=δ=0 there is no preconditioning. To symmetrize the equations we require $\beta^2 > \alpha(u^2 + v^2)$. For optimality (condition number = 1) near M=0 we need $\beta^2 = M^2$ and $\alpha = 1$. Let U be the contravariant velocity. Then for free parameters α, β, δ the eigenvalues of PA in generalized coordinates are

$$\begin{aligned} \lambda_0 &= U = uS_x + vS_y + wS_z \\ z &= 1 - \alpha + \beta^2 \\ \lambda_\pm &= \frac{zU \pm \sqrt{(z^2 - 4\beta^2)U^2 + 4\beta^2 c^2 |S|^2}}{2} \end{aligned} \tag{4}$$

δ does not affect the inviscid eigenvalues. To transition smoothly to transonic flow we demand that

$$\beta \to 1 \qquad \alpha \to 0 \qquad \delta \to 0 \quad \text{as} \quad M \to 1.$$

K_2 is typically 0.4 for inviscid cases and 1.0 for viscous cases. However, for some difficult cases K_2 can be as large as 3.0 . Since this means that the minimum cutoff is equal to or greater than the far field speed it implies that β is constant over almost the entire flow field. We wish to improve the robustness. We stress that robustness is not the same as optimality. As we approach the incompressible equations we expect the solution to have some global behavior as given by the pressure correction approach. i.e. the equations are no longer hyperbolic. Therefore, it is not surprising if β is not given locally.

Viscous Equations

We now present the preconditioned Navier-Stokes equations. Consider a symmetric positive definite preconditioner for the (constant coefficient) Navier-Stokes equations expressed as a symmetric system. We express the equations in terms of the w_N variables Using the Stokes hypothesis, $\lambda + \frac{2\mu}{3} = 0$ and ignoring the dissipation function Φ and cross derivatives, we have in two dimensions

$$\begin{aligned} &P_N^{-1}(w_N)_t + A_N(w_N)_x + B_N(w_N)_y \\ &= C_N(w_N)_{xx} + D_N(w_N)_{yy} \end{aligned}$$

or dropping the subscript N

$$\mathbf{P}^{-1} w_t + \begin{pmatrix} u & \frac{a}{\sqrt{\gamma}} & 0 & 0 \\ \frac{a}{\sqrt{\gamma}} & u & 0 & \sqrt{\frac{\gamma-1}{\gamma}}a \\ 0 & 0 & u & 0 \\ 0 & \sqrt{\frac{\gamma-1}{\gamma}}a & 0 & u \end{pmatrix} w_x$$
$$+ \begin{pmatrix} v & 0 & \frac{a}{\sqrt{\gamma}} & 0 \\ 0 & v & 0 & 0 \\ \frac{a}{\sqrt{\gamma}} & 0 & v & \sqrt{\frac{\gamma-1}{\gamma}}a \\ 0 & 0 & \sqrt{\frac{\gamma-1}{\gamma}}a & v \end{pmatrix} w_y =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{4\mu}{3\rho} & 0 & 0 \\ 0 & 0 & \frac{\mu}{\rho} & 0 \\ 0 & 0 & 0 & \frac{\gamma\mu}{Pr\rho} \end{pmatrix} w_{xx} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\mu}{\rho} & 0 & 0 \\ 0 & 0 & \frac{4\mu}{3\rho} & 0 \\ 0 & 0 & 0 & \frac{\gamma\mu}{Pr\rho} \end{pmatrix} w_{yy}$$

The Jacobian matrices are given by

$$\frac{\partial w_0}{\partial w_N} = \begin{pmatrix} \frac{1}{\sqrt{\gamma}} & 0 & 0 & \sqrt{\frac{\gamma-1}{\gamma}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{\gamma-1}{\sqrt{\gamma}} & 0 & 0 & \sqrt{\frac{\gamma-1}{\gamma}} \end{pmatrix}$$

$$\frac{\partial w_N}{\partial w_0} = \begin{pmatrix} \frac{1}{\sqrt{\gamma}} & 0 & 0 & -\frac{1}{\sqrt{\gamma}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sqrt{\frac{\gamma-1}{\gamma}} & 0 & 0 & \frac{1}{\sqrt{\gamma(\gamma-1)}} \end{pmatrix}$$

We now consider the one dimensional case in w_N variables. Then

$$\mathbf{P_N}^{-1} \frac{\partial w_N}{\partial t} + A \frac{\partial w_N}{\partial x} = C \frac{\partial^2 w_N}{\partial x^2}$$

$$A_N = \begin{pmatrix} u & \frac{a}{\sqrt{\gamma}} & 0 \\ \frac{a}{\sqrt{\gamma}} & u & \sqrt{\frac{\gamma-1}{\gamma}}a \\ 0 & \sqrt{\frac{\gamma-1}{\gamma}}a & u \end{pmatrix}$$

$$C_N = \frac{\mu}{\rho}\begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ 0 & 0 & \frac{\gamma}{Pr} \end{pmatrix}$$

We substitute ϵ in the upper left hand corner of C_N. This is done so that C_N is nonsingular and we can check the behavior as the Mach number goes to zero. The original viscous matrix is singular which makes it impossible to check its approach to a singular matrix as $M \to 0$ without using SVD.

The preconditioner, (3), is given by

$$\mathbf{P_N^{-1}} = \begin{pmatrix} \frac{1-(\delta-1)(\gamma-1)\beta^2}{\gamma\beta^2} & 0 & \frac{\sqrt{\gamma-1}[\beta^2(1-\delta)-1]}{\gamma\beta^2} \\ \frac{\alpha u}{\sqrt{\gamma}a\beta^2} & 1 & -\frac{\alpha u}{a\beta^2}\sqrt{\frac{\gamma-1}{\gamma}} \\ \frac{\sqrt{\gamma-1}[(1+(\gamma-1)\delta)\beta^2-1]}{\gamma\beta^2} & 0 & \frac{\beta^2[1+\delta(\gamma-1)]+\gamma-1}{\gamma\beta^2} \end{pmatrix}$$

This is symmetric only when $\alpha = \delta = 0$. In this case P_N reduces to

$$\mathbf{P_N} = \frac{1}{\gamma}\begin{pmatrix} \gamma - 1 + \beta_1^2 & 0 & \sqrt{\gamma-1}(\beta_2^2 - 1) \\ 0 & \gamma & 0 \\ \sqrt{\gamma-1}(\beta_2^2-1) & 0 & 1 + (\gamma-1)\beta_3^2 \end{pmatrix} \tag{5}$$

Note, that we have replaced β by three different β_i. Expressed in w_0 variables this is

$$\mathbf{P_0} = \begin{pmatrix} p_{11} & 0 & p_{13} \\ 0 & 1 & 0 \\ p_{31} & 0 & p_{33} \end{pmatrix}$$

$$\begin{aligned} p_{11} &= \frac{\beta_1^2 + 2(\gamma-1)\beta_2^2 + (\gamma-1)^2\beta_3^2}{\gamma^2} \\ p_{13} &= \frac{\beta_2^2 - \beta_1^2 + (\gamma-1)(\beta_3^2 - \beta_2^2)}{\gamma^2} \\ p_{31} &= (\gamma-1)p_{13} \\ p_{33} &= \frac{\gamma^2 + (\gamma-1)(\beta_1^2 - 2\beta_2^2 + \beta_3^2)}{\gamma^2} \end{aligned}$$

It then follows that

$$\mathbf{P_N}C_N = \frac{\mu}{\rho}\begin{pmatrix} \frac{\gamma-1+\beta_1^2}{\gamma}\epsilon & 0 & \frac{\sqrt{\gamma-1}}{Pr}(1-\beta_2^2) \\ 0 & \frac{4}{3} & 0 \\ \frac{\sqrt{\gamma-1}}{\gamma}(1-\beta_2^2)\epsilon & 0 & \frac{1+(\gamma-1)\beta_3^2}{Pr} \end{pmatrix}$$

and

$$\det(\mathbf{P_N}C_N) = \frac{4\mu\epsilon}{3\rho\gamma Pr} \left(\beta_1^2 + 2(\gamma-1)\beta_2^2 + (\gamma-1)^2\beta_3^2 + (\gamma-1)(\beta_1^2\beta_3^2 - \beta_2^4)\right).$$

In w_0 variables

$$A_0 = \begin{pmatrix} u & a & 0 \\ a & u & 0 \\ 0 & 0 & u \end{pmatrix} \quad C_0 = \frac{\mu}{\rho}\begin{pmatrix} \frac{\gamma-1}{Pr} & 0 & \frac{1}{\rho a Pr} \\ 0 & \frac{4}{3} & 0 \\ \frac{(\gamma-1)\rho a}{Pr} & 0 & \frac{1}{Pr} \end{pmatrix}$$

We consider the preconditioner with all the β_i equal.

$$\mathbf{P_0} = \begin{pmatrix} \beta_1^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Consider finite differences based on central differences. Let G be the symbol of the space approximation. We Fourier transform with Fourier variable k and $\theta = k\Delta x$. Then

$$G(\theta) = iAg_I - C\frac{g_r}{\Delta x},$$
$$g_I = \frac{\sin(\theta)}{\Delta x}, \qquad g_r = \frac{2(1-\cos(\theta))}{\Delta x}$$

Since G is not a normal matrix the eigenvalue analysis is not sufficient for stability. Nevertheless, it indicates the properties of the system. Furthermore, the system is only of size three and it is diagonalizable except for a small number of frequencies. One obtains a cubic equation for the eigenvalues of the G. If $Pr = \frac{3}{4}$ we can factor the equation explicitly and find that one eigenvalue is given by

$$\lambda_0 = iug_I - \frac{4\mu}{3\rho}\frac{g_r}{\Delta x} \quad .$$

Define

$$\alpha_2 = \frac{4\mu}{3\rho} \quad \alpha_3 = \frac{\gamma\mu}{\rho Pr} = \gamma\alpha_2.$$

We obtain a quadratic equation for the acoustic eigenvalues $\lambda_\pm$:

$$\lambda^2 - \lambda\left[\beta^2(\gamma-1)(\delta-1)-1\right]\alpha_2 g_r + iug_I(1-\alpha+\beta^2) + \beta^2\left[(a^2-u^2)g_I^2 - iug_I\alpha_3\frac{g_r}{\Delta x}\right] = 0.$$

Solving the quadratic equation and expanding the solution for $\frac{R_\Delta\beta}{M}$small we have three (approximate) eigenvalues for G.

$$\begin{aligned} \frac{\Delta x}{\alpha_2}\lambda_0 &= -g_r + iR_\Delta g_I \\ \frac{\Delta x}{\alpha_2}\lambda_+ &= -g_r + R_\Delta^2\frac{\beta^2}{M^2}\frac{g_I^2}{g_r} + i\gamma\beta^2 R_\Delta g_I \\ \frac{\Delta x}{\alpha_2}\lambda_- &= -R_\Delta^2\frac{\beta^2}{M^2}\frac{g_I^2}{g_r} - i\gamma\beta^2 R_\Delta g_I \end{aligned}$$

As R_Δ goes to zero λ_- approaches zero. This is to be expected since the matrix C has a zero eigenvalue. We wish to choose β to equalize these eigenvalues. By our assumption $R_\Delta^2\frac{\beta^2}{M^2} << 1$ and so the real part of λ_0 and λ_+ are equalized. To equalize the imaginary part we require $\beta^2 R_\Delta << 1$. Hence, with one free parameter β we cannot equalize both the real and imaginary parts of the eigenvalues as the cell Reynolds number approaches zero.

Our experience has been that it is necessary to have a a high value for the cutoff. Hence, one cannot incorporate a viscous correction into the local Mach number since in any case the global minimum usual applies[4–6,15] . Instead, we first calculate an inviscid value for β including all cutoffs and only afterwards include a viscous correction. This implies that when the inflow Mach number is not extremely low that β is not that small. Hence, we cannot use the cell Reynolds number raised to a high power as this would quickly bring β to its maximum value of 1. Instead, to account for viscous effects we choose

$$\beta^2 = (1 + 2R_\Delta^{-1/3})\beta^2_{\text{inv}} \qquad \text{and} \quad \beta^2 \le 1 \tag{6}$$

The definition of a cell Reynolds number for multidimensions with a general curvilinear mesh is not clear. In particular it should not be done dimension by dimension since physically the boundary layer normal viscous forces are balanced by a tangential inviscid force. We choose

$$R_\Delta = \frac{\max \lambda_{\text{vis}}}{\max \lambda_{\text{inv}}}$$

where the maximum, in the numerator and denominator, is over all the dimensions independently.

Conservation Variables

The primitive temperatire variables w_T have the advantage, for low speed flow, of being appropriate for incompressible flow. However, all sets of variables except for the conservation variables, w_c, have the disadvantage that even when the preconditioning is turned off, i.e. $P = I$, the algorithm is not equivalent to the non-preconditioned one, because of the transformation of variables in the artificial viscosity. Hence, we consider the algorithm with the preconditioning done in conservation variables. The preconditioner P is now a dense matrix. However, because of its structure it is still possible to compute the product of P and a vector efficiently.[11] Let A be the Jacobian matrix in a generalized x direction with covariant velocity $U = uS_x + vS_y + wS_z$. To calculate the matrices needed for the artificial viscosity we do it in two stages. We first calculate $|PA|$ in, w_0, entropy variables. $|PA| = T|D|T^{-1}$ where the columns of T are the eigenvectors and D is a diagonal matrix containing the absolute value of the eigenvectors plus cutoffs to prevent them from becoming too small near stagnation points. These diagonal elements also include cell aspect ratio information. To transform from w_0 to w_c variables consider the one dimensional equation

$$\frac{\partial w_0}{\partial t} + A_0 \frac{\partial w_0}{\partial x} = \frac{\partial (|PA|_0 \frac{\partial w_0}{\partial x})}{\partial x}$$

but $\frac{\partial w_0}{\partial x} = \frac{\partial w_0}{\partial w_c}\frac{\partial w_c}{\partial x}$ so

$$\frac{\partial w_0}{\partial w_c}\frac{\partial w_c}{\partial t} + A_0 \frac{\partial w_0}{\partial w_c}\frac{\partial w_c}{\partial x} = \frac{\partial (|PA|_0 \frac{\partial w_0}{\partial w_c}\frac{\partial w_c}{\partial x})}{\partial x}$$

Multiplying the equation on the left by $\frac{\partial w_c}{\partial w_0}$ we get

$$\frac{\partial w_c}{\partial t} + A_c \frac{\partial w_c}{\partial x} = \frac{\partial w_c}{\partial w_0}\frac{\partial (|PA|_0 \frac{\partial w_0}{\partial w_c}\frac{\partial w_c}{\partial x})}{\partial x}$$

with $A_c = \frac{\partial w_c}{\partial w_0} A_0 \frac{\partial w_0}{\partial w_c}$.

If we can bring $\frac{\partial w_c}{\partial w_0}$ into the last derivative we see that the artificial viscosity transforms as

$$|PA|_c = \frac{\partial w_c}{\partial w_0}|PA|_0 \frac{\partial w_0}{\partial w_c}$$

However, this is strictly valid only for constant coefficients. We shall use this formula to derive an artificial viscosity for different variables. However, these are no longer strictly equivalent for nonlinear problems.

The transformation matrices connecting the primitive and conservation variables are:

$$\frac{\partial w_0}{\partial w_c} = \begin{pmatrix} \frac{\frac{\gamma-1}{2}q^2}{\rho a} & \frac{(1-\gamma)u}{\rho a} & \frac{(1-\gamma)v}{\rho a} & \frac{\gamma-1}{\rho a} \\ \frac{-u}{\rho} & \frac{1}{\rho} & 0 & 0 \\ \frac{-v}{\rho} & 0 & \frac{1}{\rho} & 0 \\ \frac{\gamma-1}{2}q^2 - a^2 & (1-\gamma)u & (1-\gamma)v & \gamma-1 \end{pmatrix}$$

$$\frac{\partial w_c}{\partial w_0} = \begin{pmatrix} \frac{\rho a}{a^2} & 0 & 0 & \frac{-1}{a^2} \\ \frac{\rho a u}{a^2} & \rho & 0 & \frac{-u}{a^2} \\ \frac{\rho a v}{a^2} & 0 & \rho & \frac{-v}{a^2} \\ \rho a(\frac{1}{\gamma-1} + \frac{M^2}{2}) & \rho u & \rho v & \frac{-M^2}{2} \end{pmatrix}.$$

$|\mathbf{P}A|_0$ only appears as a matrix multiplying a vector. The eigenvalues λ of $|PA|$ are given by (4). We consider only the case $\alpha = 0$ in (3). To calculate $z = |PA|x$, $x = (x_1, ..., x_5)^t$ we define

$$\begin{aligned}
|S|^2 &= S_x^2 + S_y^2 + S_z^2 \\
Y &= \frac{S_x x_2 + S_y x_3 + S_z x_4}{|S|^2} \\
t &= \frac{\delta U}{(\lambda_+ - \lambda_-)\beta^2 c^2 |S|^2} \\
Q_1 &= (\lambda_+ - U)(\lambda_- - U)(|\lambda_+| - |\lambda_-|)t \\
\Lambda_1 &= |\lambda_-|(\lambda_+ - U) - |\lambda_+|(\lambda_- - U) - |\lambda_0|(\lambda_+ - \lambda_-) \\
\Lambda_2 &= \frac{c(|\lambda_+| - |\lambda_-|)x_1 + \Lambda_1 Y}{\lambda_+ - \lambda_-} + \Lambda_1 t x_5
\end{aligned}$$

Then

$$\begin{aligned}
z_1 &= \frac{(|\lambda_+|(\lambda_+ - U) - |\lambda_-|(\lambda_- - U))x_1 + \beta^2 c(|\lambda_+| - |\lambda_-|)Y}{\lambda_+ - \lambda_-} \\
&\quad + Q_1 x_5 \\
z_2 &= |\lambda_0| x_2 + S_x \Lambda_2 \\
z_3 &= |\lambda_0| x_3 + S_y \Lambda_2 \\
z_4 &= |\lambda_0| x_4 + S_z \Lambda_2 \\
z_5 &= |\lambda_0| x_5
\end{aligned}$$

We generally find that for the optimal parameters the convergence rate for the preconditioning in conservation variables is a little slower than in w_T variables. This can frequently be improved by using a slightly larger β with the preconditioning in w_c variables. Alternatively, one can increase the residual smoothing paramter.

With this set of variables choosing $\beta = 1$ and $\alpha = \delta = 0$ in the preconditioning we recover the nonpreconditioned code with $P = I$ and the original nonpreconditioned matrix viscosity. Hence, this set of variables is particularly well suited for a simultaneously combined preconditioned-nonpreconditioned scheme. The preconditioning we have described is appropriate for low speed flow. Hence, it is reasonable to remove the preconditioning when the Mach number is no longer very low. Experience shows that preconditioning is helpful when the Mach number is below the range of 0.2 to 0.3. Hence, we propose to modify β^2 to eliminate the preconditioning when we reach this intermediate zone.

$$\beta^2 = \min(\beta_{\rm orig} f(M), 1)$$

$f(0) = 1$ and $f(M) < f_1 \sim 1.1$ when $M < M_1 \sim 0.3$ and $f(M) = f_2$ is large when $M > M_2 \sim 0.5$. We choose $f(M)$ linearly between these knots so that $f(M)$ will be monotone. Hence, for $M > M_2$ there is no preconditioning. Since, we are using conservation variables we recover the original algorithm.

Computational Results

Method

We use a central difference scheme with a matrix valued viscosity to accelerate the flow to a steady state. The scheme is advanced in pseudo-time by a five stage Runge-Kutta method. The coefficients of the Runge-Kutta method is slightly nonstandard and are $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{2}$, $\frac{1}{2}$, 1. A matrix artificial viscosity is used.[8] The artificial viscosity is updated on the odd stages without any averaging with previous values. Residual smoothing and multigrid are used to further accelerate the convergence to a steady state.[3,16]

Flapped airfoil

We consider steady viscous turbulent flow around an airfoil with a flap. To resolve the flap leading edge a refined mesh is used in that region, see Fig. (1). This gives rise to high aspect ratios in the direction normal to the boundary layer. Hence, the code needs to handle high aspect ratios in both directions. The mesh is an C mesh with 185×65 points. A one equation Spalart-Allmaras turbulence model is used. This is solved in pseudo-time by an ADI solver. The time step for the turbulence model is 5 times the local time step of the pseudo-time step of the fluid equations The inflow conditions are $M_\infty = 0.2$, $Re = 2.76 \times 10^6$ and angle of attack 12°. For preconditioning $\beta_{min} = 1.0$ and $\alpha = 1$. In figure 2 we see the improvement in the residual given by the viscous correction.

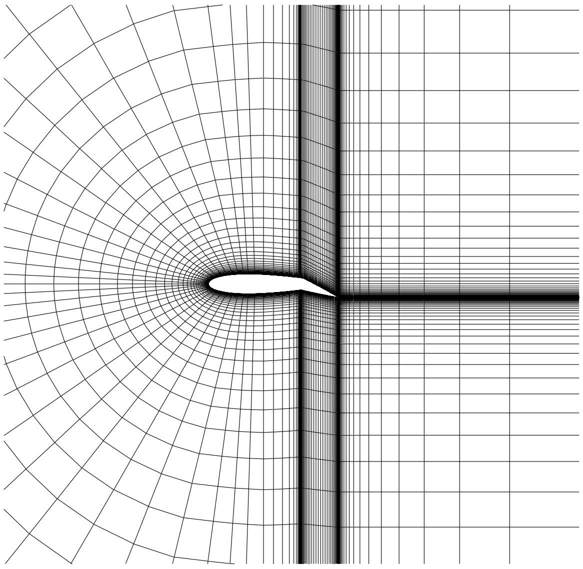

Fig. 1 grid for flapped airfoil

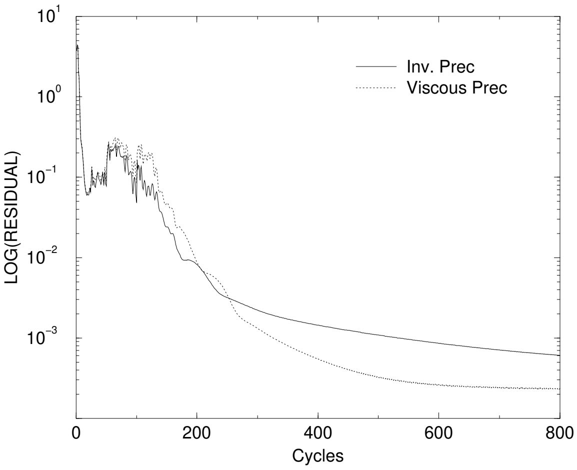

Fig. 2 flapped airfoil - residual history

NACA4412

We next consider turbulent flow around a NACA4412 airfoil. The mesh is an C mesh with 257 × 81 points and is displayed in Fig. (3). A Baldwin-Lomax turbulence model is used. The inflow conditions are $M_\infty = 0.2$, $Re = 1.52 \times 10^6$ with an angle of attack of 13.87°.

We will compare the convergence rate without preconditioning versus several different preconditionings. For all cases we use $\beta_{min} = 1.0$. iprec=2 denotes the preconditioning based on the pressure variables, w_T. In this case we consider both $\alpha = 1$ and $\alpha = 0$. iprec=1 denotes the preconditioning using conservation variables. In this case $\alpha = 1$ diverges and we only show $\alpha = 0$. In figures 4 , 5 and 6 we compare the residual as well as the lift and drag history. There are 50 iterations on two coarser grids and 500 RK iterations on the finest grid. We see that the residual converges fastest with the preconditioning based on pressure-temperature variables. $\alpha = 1$ is a little faster than $\alpha = 0$ though it stalls at the end. Preconditioning based on conservation variables is considerably slower while without preconditioning the residual converges very slowly. In all these cases the residual is plotted for the first variable. This is the density residual for conservation variables and the pressure residual for w_T variables. However, further investigation shows similar convergence rates for all residuals.

When we examine the lift and drag histories the story is a little different. It is clear that without preconditioning that the lift and drag converge slower than with preconditioning. Pressure variables with $\alpha = 1$ again give the fastest convergence rate thgough the difference is small. When $\alpha = 0$ both sets of variables now give similar convergence rates. For time dependent problems using a dual time step, not displayed here, the conservation variables gave a faster convergence for lift and drag even though the residual convergence rate was slower. Hence, the convergence of residuals and global quantities do not always behave in a similar fashion when changing the basic variables used in the algorithm. We note that the final drag differes between the preconditioning and non-preconditioned computations. This is to be expected since the preconditioning changes the artificial viscosity and hence the steady state.[12]

Celestina and Turkel[1] show that for internal flows, both rotating and non-rotating turbomachinery, that the mass flow and total temperature converge much faster for low Mach number flows when using preconditioning. For these flows the residuals never converge more than two or three orders of magnitude.

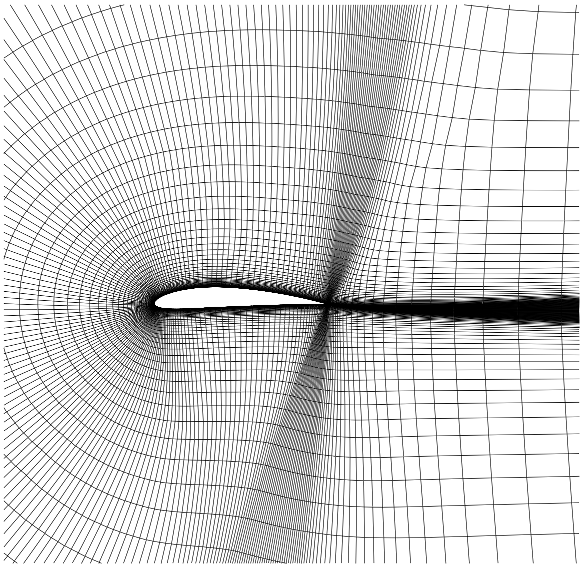

Fig. 3 grid for NACA4412

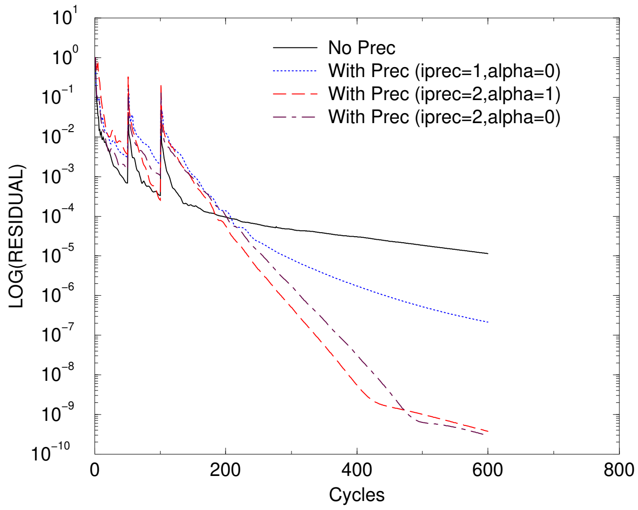

Fig. 4 NACA4412 - residual history

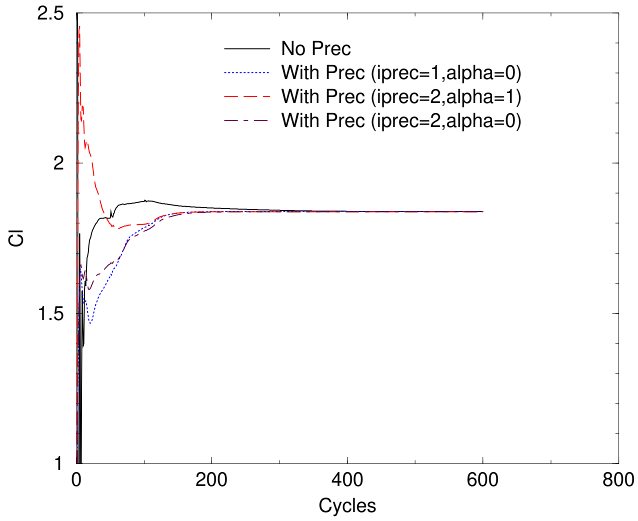

Fig. 5 NACA4412 - lift history

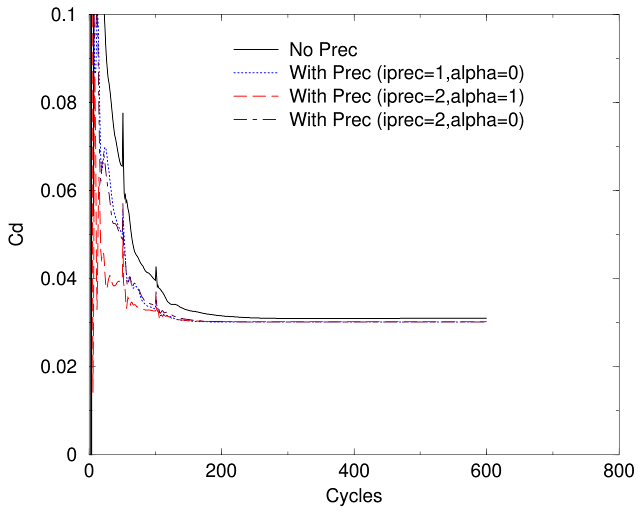

Fig. 6 NACA4412 - drag history

Douglas three element airfoil

We consider steady turbulent flow around a three element airfoil . The mesh consists of 20 blocks and is shown in (7). An one equation SA turbulence model is used. The Mach and Reynolds numbers are $M_\infty = 0.2$, $Re = .40909 \times 10^6$. The angle of attack is $16.2°$. For preconditioning, only $\beta^2_{min} = 3.0$ and $\delta = 1$ worked. This is still the major drawback to robustness. However, we could still use $\alpha = 1$. In figure 8 we show the improvement in the residual history due to the viscous correction to β.

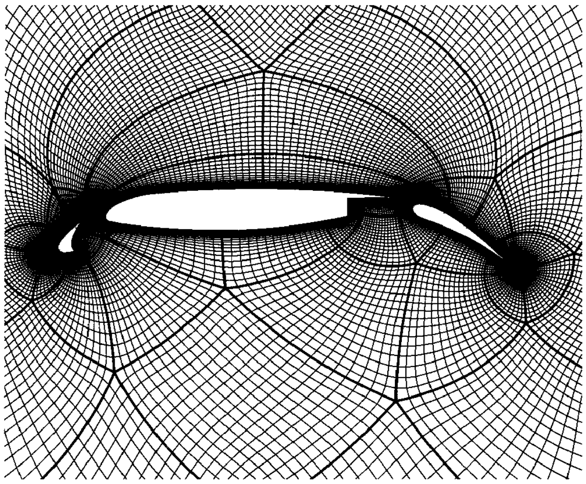

Fig. 7 grid for 3 element Douglas airfoil

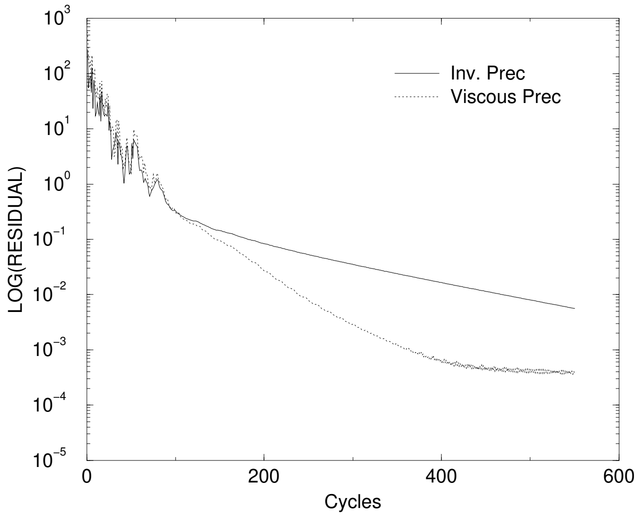

Fig. 8 3 element Douglas airfoil - residual history

NACA0012

We consider viscous turbulent flow around a NACA0012. The grid is an O mesh with 141×61 points and is shown in Fig. 9. A one equation Spalart-Allmaras turbulence model is used. This is solved using an ADI method with either 1 or 2 subiterations per fluid time step. The inflow conditions are $M_\infty = 0.2$ $Re = 3 \times 10^6$. For preconditioning $\beta_{min} = 1.0$. When the angle of attack is sufficiently high the flow becomes time dependent. We consider the case that $\alpha = 15°$ which is close to the critical angle when the steady flow is no longer stable.

The number of subiterations in the turbulence model has a dramatic effect on the solution. In particular we have found that it sometimes determines whether the numerical solution is steady or unsteady. It is not clear why more subiterations seems to make the numerical solution less physical. The residual and lift-drag do not always behave in a similar fashion. Ultimately this case should be solved with a time dependent algorithm, Then it is questionable to decide how to adaptively end the subiterations when the residual and lift do not behave in a consistent manner. Standard w_T preconditioning gives a much better residual history than no preconditioning.

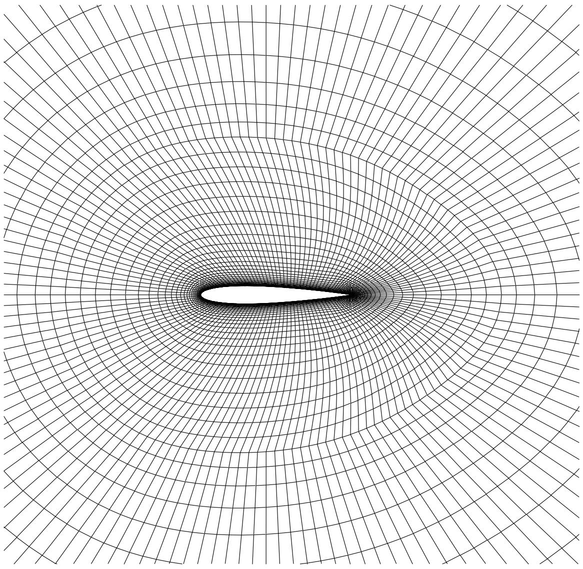

Fig. 9 grid for NACA0012

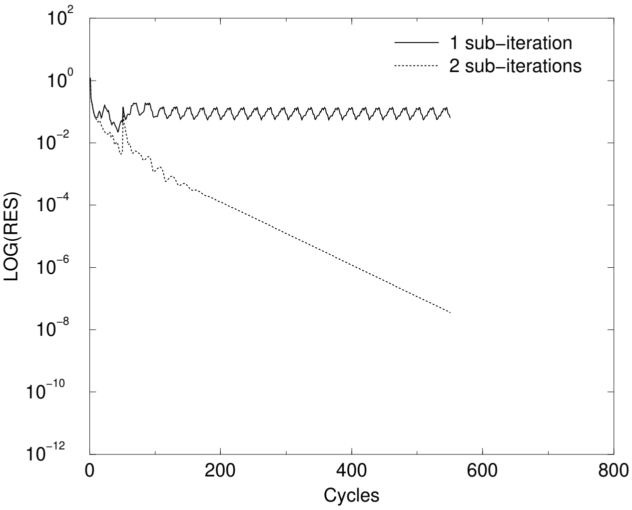

Fig. 10 NACA0012 - turbulent -residual history

Trapezoidal Wing

We consider steady viscous turbulent flow around a trapezoidal wing together with a flat and slat. The mesh consists of 48 blocks with patched grids at the block interfaces, see fig. (11). The inflow conditions are $M_\infty = 0.2$, $Re = .1083 \times 10^6$, $\alpha = 14.0°$. An one equation Spalart-Allmaras turbulence model is used. For preconditioning we choose $\beta_{min} = 1.0$ and $\delta = 1$. We note, that the difficulties that appeared with the three element airfoil requiring an increase to $\beta_{min} = 1.7$ did not appear even though this case is in a generalized sense a three dimensional extension of that configuration. For this case the viscous correction to β slows convergence. Though the preconditioning does not help the convergence of the residual it does accelerate the convergence of the lift and drag, see fig. (12,13). This again confirms our experience that the residual convergence does not always indicate what is happening to the global quantities like lift and drag for external flows and mass flow and total temperature for internal flows.

Conclusion

We consider the effect of preconditioning as the Mach number goes to zero where we have included viscous effects. This shows that the parameter β needs to be a function of the local cell Reynolds number. However, one can choose several extensions of the parameter β. These are equal in the inviscid limit but behave differently in the viscous limit. We also introduced a way to construct a cell Reynolds number for general meshes. We also considered ways of turning off the preconditioning at intermediate Mach numbers. This might alleviate some of the robustness problems since now the preconditioning is only used for very low Mach numbers. We also considered preconditioning using both pressure-temperature variables and conservation variables. The pressure variables give a better convergence rate for the residual but not as great a gain for the lift and drag. We demonstrate these effects in several computations of high lift problems.

Fig. 11 grid for trapezoidal wing

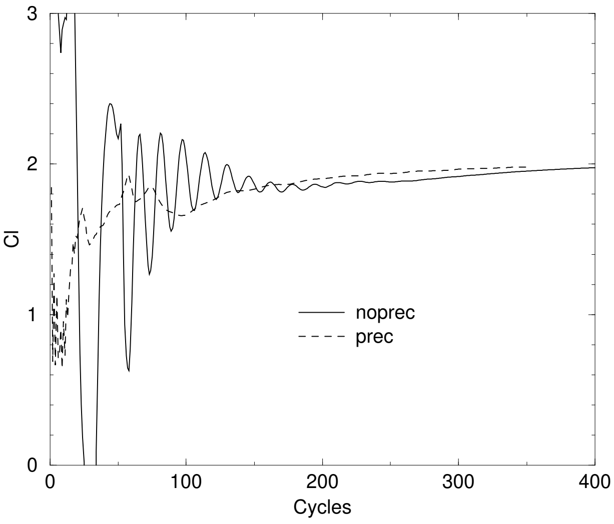

Fig. 12 C_L for trapezoidal wing

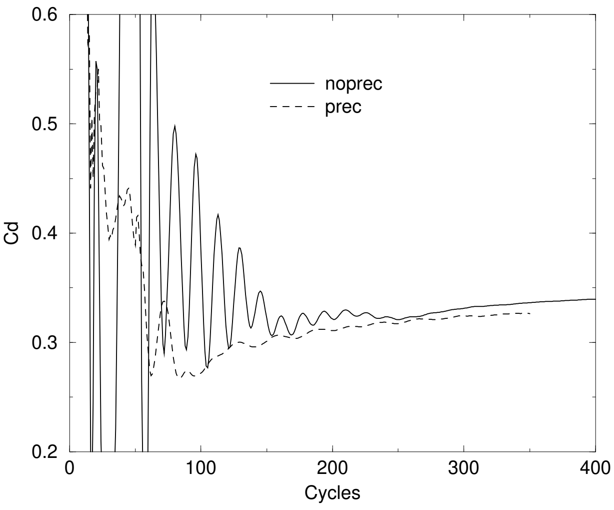

Fig. 13 C_D for trapezoidal wing

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